The Laws of Systems Science and Engineering: have we progressed the last 20 years?

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IDSS
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- **Has Systems Engineering progressed at all in the last 20 years?**
- **What is the theoretical (scientific) basis of Systems Engineering?**
- **What are the Laws of Systems Science? What is the 1st Law?**

Fundamental Laws in Science

- First Law of Thermodynamics
  - Conservation of Energy
  - Rudolf Clausius 1850

$$\Delta U = Q - W.$$

- Second Law of Classical Mechanics
  - Conservation of Angular Momentum
  - Leonhard Euler 1736

$$\dot{\underline{H}} = \underline{T} - \underline{\omega} \times \left[\underline{\underline{I}}\, \underline{\omega} \right]$$

- ***What is the conserved quantity in Systems Science?***

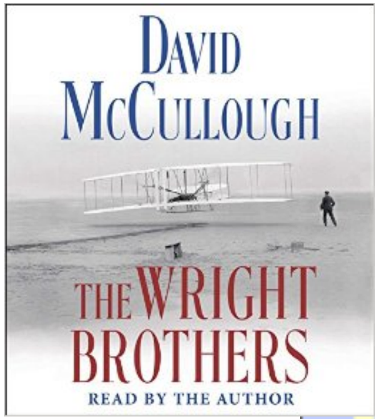
COMPLEXITY !

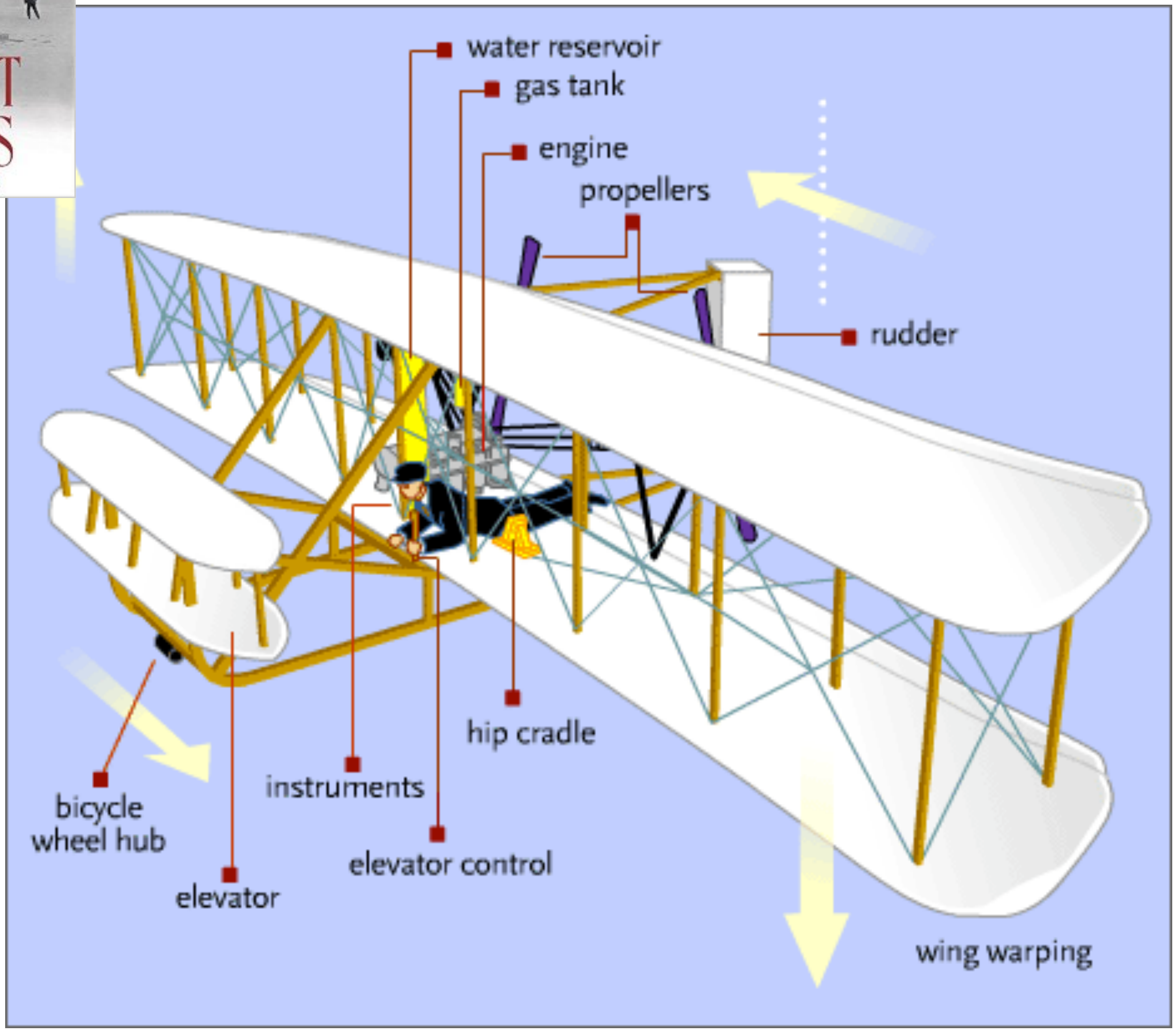
Why should we care about complexity?

How do we quantify complexity?

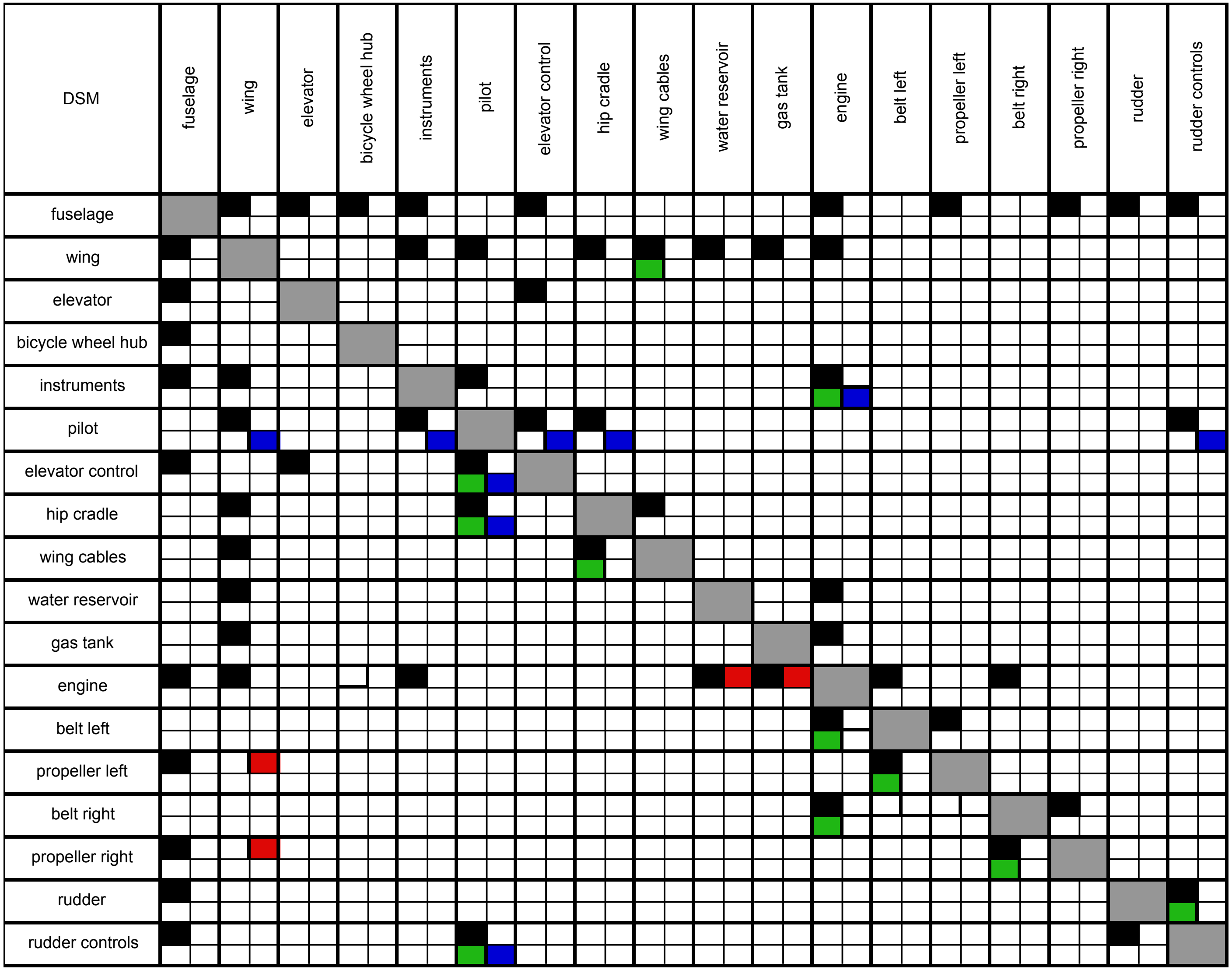
The First Law of Systems Science ?

The Wright Flyer (1903)





Structural DSM of Wright Flyer



Legend
- Physical connection
- Mass flow
- Energy flow
- Information flow

DSM 18x18

<u>Connections</u>

62 Physical

4 Mass Flow

11 Energy Flow

9 Info Flow

Total: 86

NZF = 86/1,224

= **7% density**

<k>=~5

Design Structure Matrix (DSM) – captures structure of elements of form

Augustine's 16th Law

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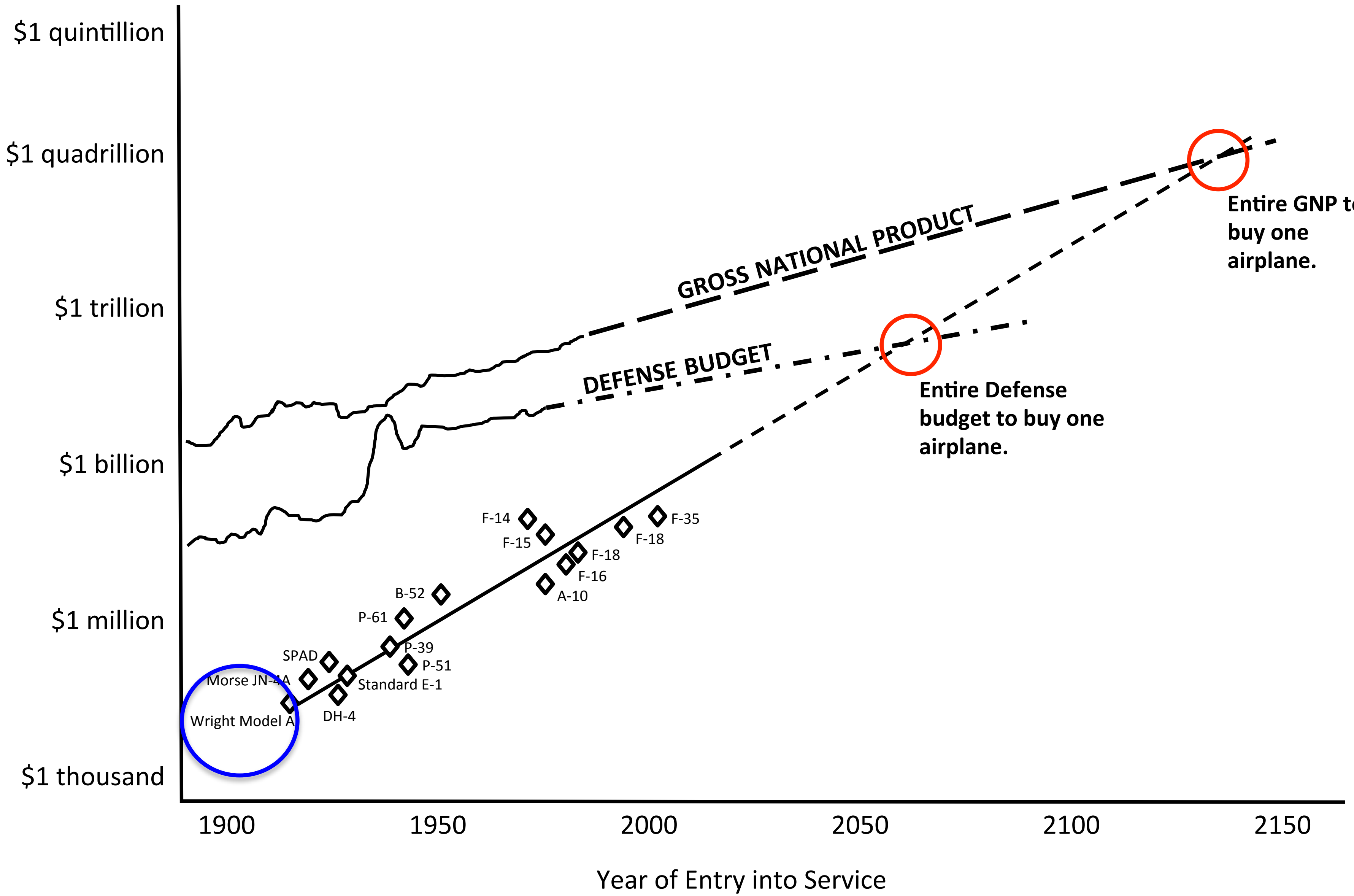
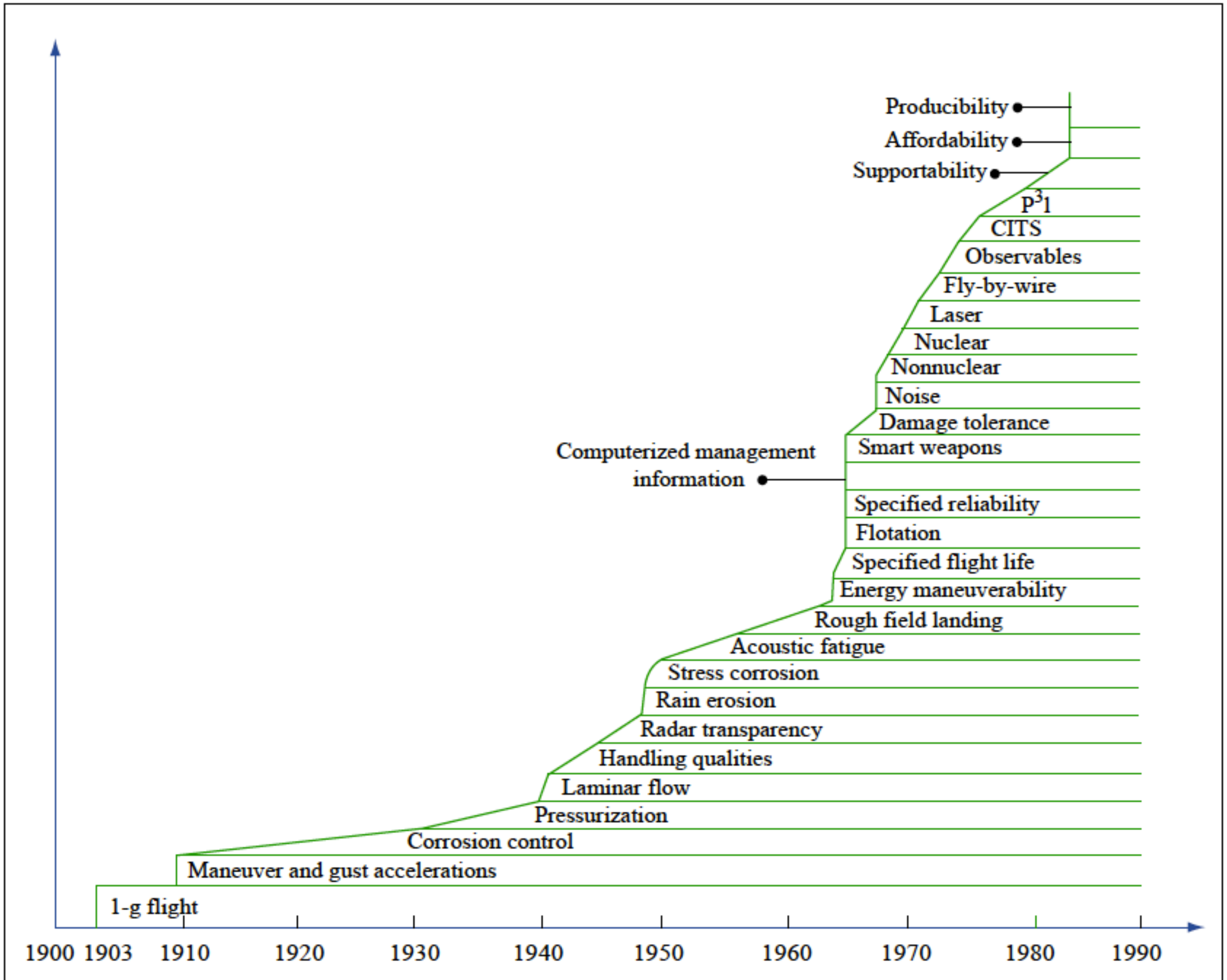


Norm Augustine, *Augustine's Laws*, 6th Edition, AIAA Press, 1997.

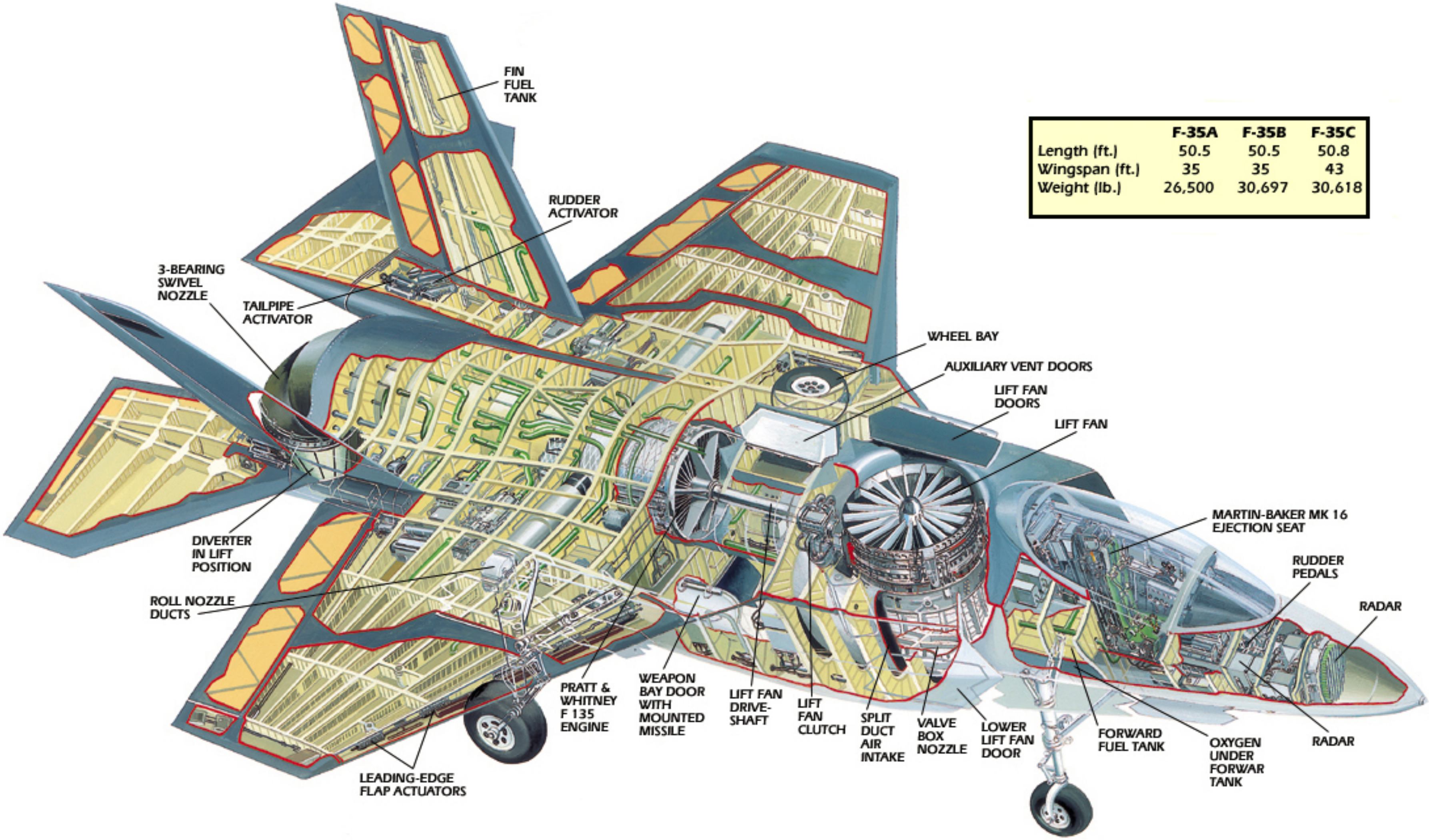
Functional Requirements Explosion in Aviation



Design requirements growth for aerospace vehicles.

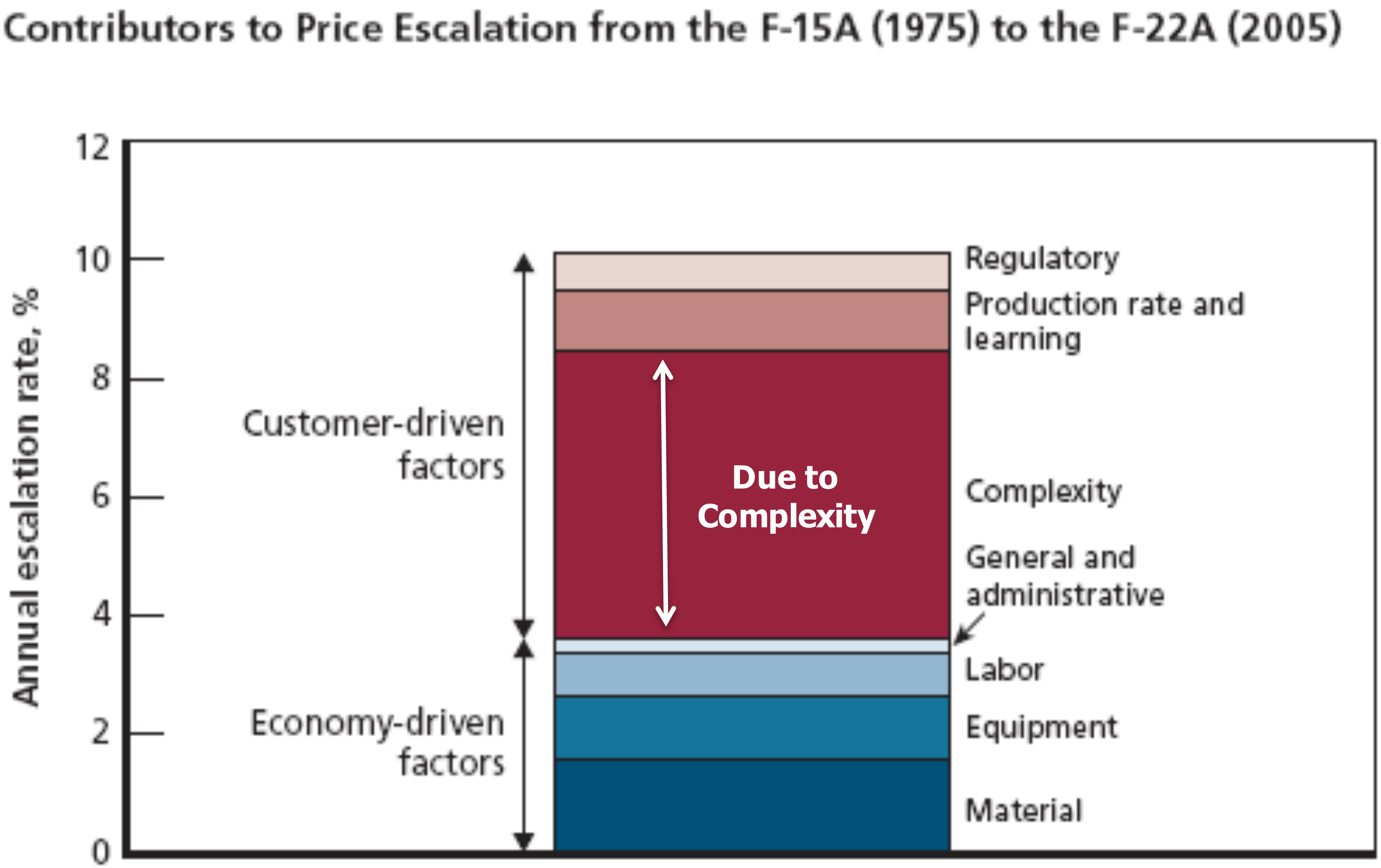
Image by MIT OpenCourseWare.

F-35 JSF



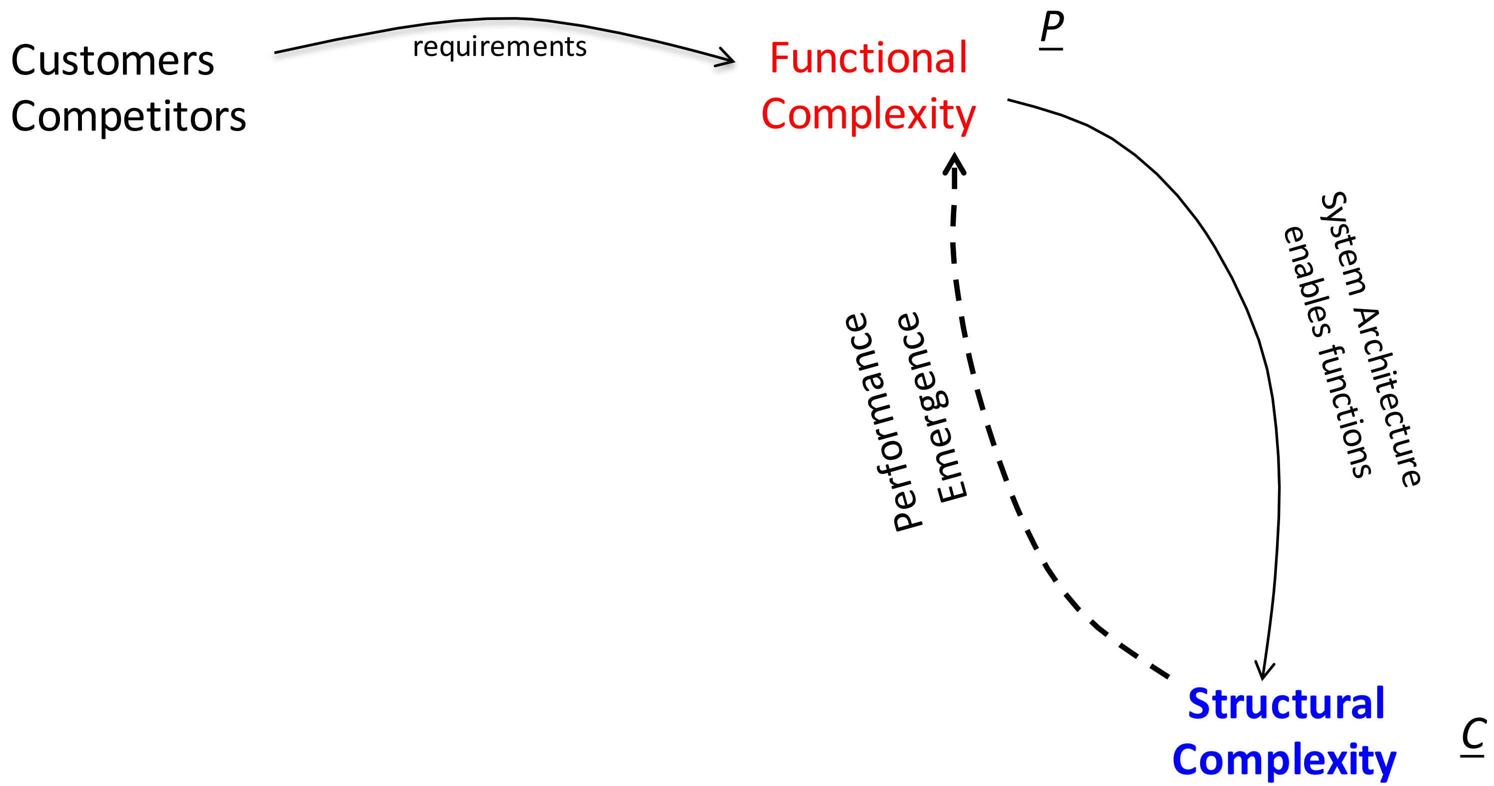
	F-35A	F-35B	F-35C
Length (ft.)	50.5	50.5	50.8
Wingspan (ft.)	35	35	43
Weight (lb.)	26,500	30,697	30,618

What is driving this escalation of cost?



Source: DARPA TTO (2008)

Two Dimensions of Complexity



Why should we care about complexity?

How do we quantify complexity?

The First Law of Systems Science?

The Structural Complexity Metric

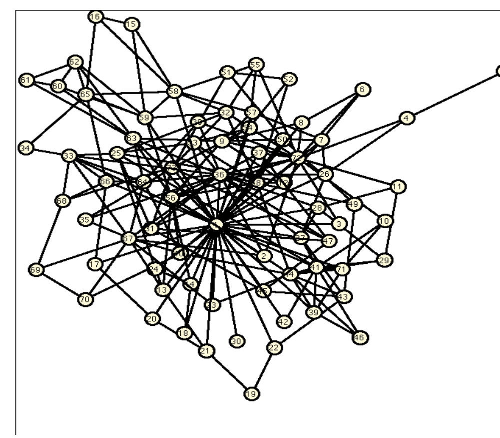
Structural Complexity, $C = C_1 + C_2 . C_3$

This functional form inspired by the solution of the *steady-state Schrodinger equation* of organic molecular systems [Gutman 1978, 2000].

Complexity due to components alone (number and heterogeneity of components)

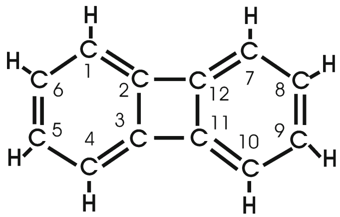
Complexity due to pair-wise component interactions (number and heterogeneity of interactions)

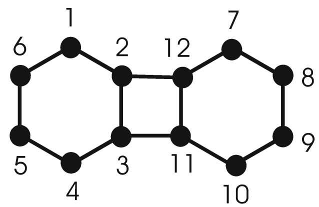
Complexity due to topological formation (a scaling factor) – due to dependency structure



Sinha, Kaushik, and Olivier L. de Weck. "A network-based structural complexity metric for engineered complex systems." In *Systems Conference (SysCon), 2013 IEEE International*, pp. 426-430. IEEE, 2013.

System Hamiltonian and Complexity





$$[\mathbf{H}]_{ij} = \begin{cases} \alpha & \text{if } i = j \\ \beta & \text{if the atoms } i \text{ and } j \text{ are chemically bonded} \\ 0 & \text{if there is no chemical bond between the atoms } i \text{ and } j. \end{cases}$$

$$\mathbf{H} = \alpha \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix} + \beta \begin{bmatrix} 0&1&0&0&0&1&0&0&0&0&0&0 \\ 1&0&1&0&0&0&0&0&0&0&0&1 \\ 0&1&0&1&0&0&0&0&0&0&1&0 \\ 0&0&1&0&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&1&0&0&0&0&0&0 \\ 1&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&1 \\ 0&0&0&0&0&0&1&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1&0&1&0 \\ 0&0&1&0&0&0&0&0&0&1&0&1 \\ 0&1&0&0&0&0&1&0&0&0&1&0 \end{bmatrix}.$$

$$H = \alpha I_n + \beta A(G)$$

$$H\psi = \varepsilon\psi$$

$$|\varepsilon_i| = \alpha + \beta\sigma_i;\ \varepsilon_\pi = \sum_{i=1}^{n} h_i |\varepsilon_i|$$

$$\varepsilon_\pi = n\alpha + \beta \sum_{i=1}^{n} h_i \sigma_i \le n\alpha + \beta \underbrace{\left(\sum_{i=1}^{n} h_i\right)}_{n} \underbrace{\left(\sum_{i=1}^{n} \sigma_i\right)}_{E(A)}$$

$$\therefore \varepsilon_\pi \le n\alpha + n^2\beta\left(\frac{E(A)}{n}\right)$$

Introduce the notion of *configuration energy*:

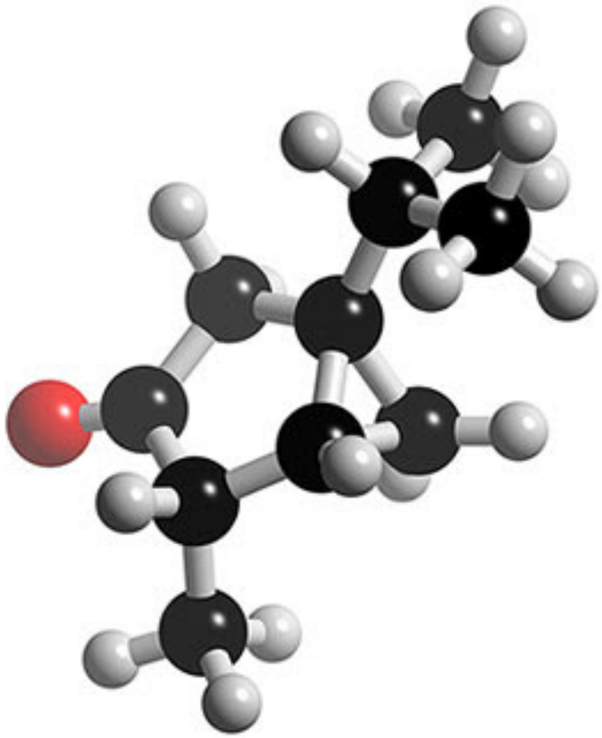
$$\Xi := \underbrace{n\hat{\alpha}}_{C_1} + \underbrace{m\hat{\beta}}_{C_2} \underbrace{\left(\frac{E(A)}{n}\right)}_{C_3} = C_1 + C_2 C_3$$

Use the above functional form to measure the complexity associated to the system structure – ***Structural Complexity*** of the system where α's stand for component complexity while β's stand for interface complexity:

$$C = C_1 + C_2 C_3$$

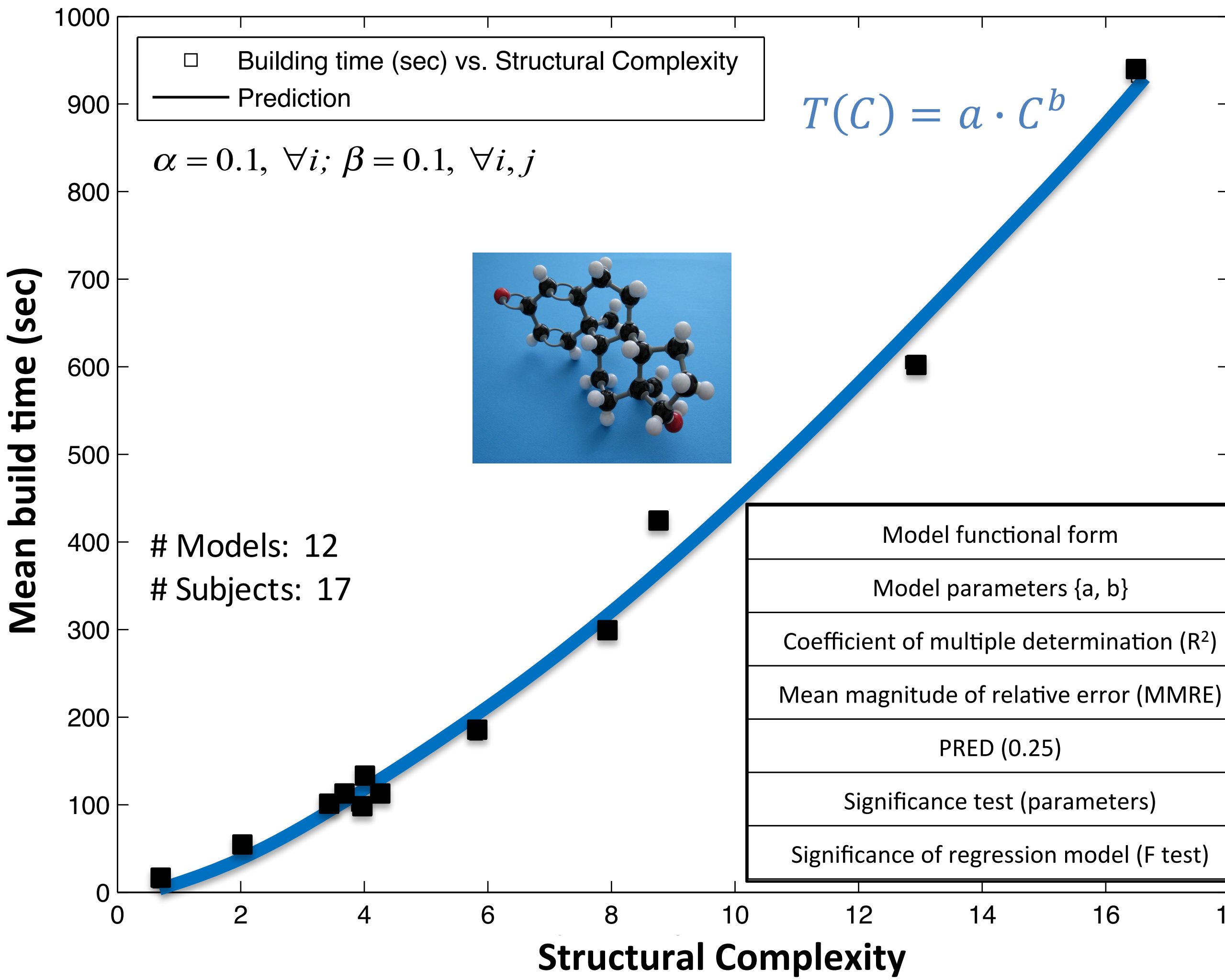
$$= \sum_{i=1}^{n} \alpha_i + \left(\sum_{i=1}^{n}\sum_{j=1}^{n} \beta_{ij}\right)\left(\frac{E(A)}{n}\right) = \sum_{i=1}^{n} \alpha_i + \left(\sum_{i=1}^{n}\sum_{j=1}^{n} \beta_{ij}\right)\gamma E(A)$$

Human Cognitive Experiments



Molecule #10

Humans slows down with complexity



Exponent
b=~ 1.5

Model functional form	$Y = aX^b$
Model parameters {a, b}	{ 14.68, 1.4775}
Coefficient of multiple determination (R^2)	0.992
Mean magnitude of relative error (MMRE)	0.107
PRED (0.25)	0.9167
Significance test (parameters)	$t_a = 28.2$, $t_b = 30.67$ ($>t_0 = 2.131$)
Significance of regression model (F test)	$f = 124 > f_{0.05,1,10} = 4.54$

$$\textit{Structural Complexity},\ C = O(n^{1.08}) \leftarrow \textit{mild super-linearity}$$

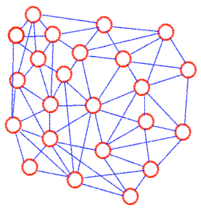
$$\textit{Average build time},\ t = O(C^{1.48}) \leftarrow \textit{strong super-linearity}$$

Why should we care about complexity?

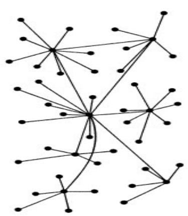
How do we quantify complexity?

The First Law of Systems Science?

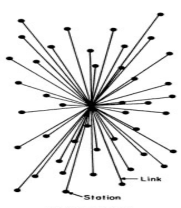
Topological Complexity C_3: Important Properties



"Distributed" Architecture



"Hierarchical" Architecture



Centralized architecture

Simple components / constituents / building blocks with intricate connectivity structure

Higher system integration effort

Increasing Topological Complexity (C_3)

Complex components / constituents / building blocks with simple connectivity structure

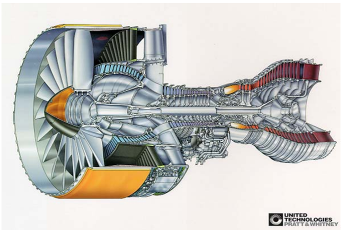
Lower system integration effort

$$Centralized\ Architecture \rightarrow hypoenergetic,\ C_3 < 1$$

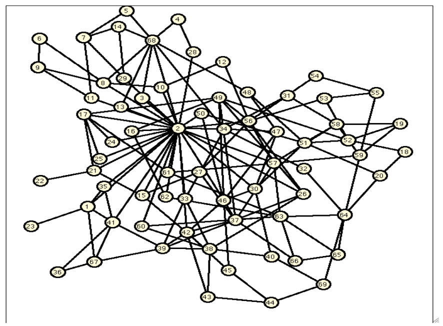
$$Hierarchical\ /\ layered\ Architecture \rightarrow transitional,\ 1 \le C_3 < 2$$

$$Distributed\ Architecture \rightarrow hyperenergetic,\ C_3 \ge 2$$

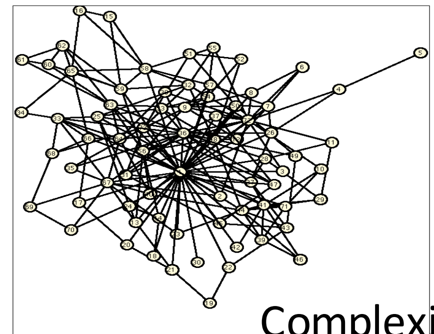
Complexity Increase of Aircraft Engines

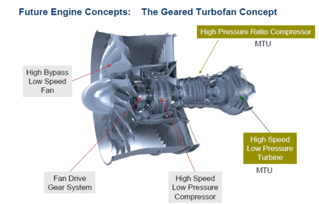


Old



Complexity = 351





New

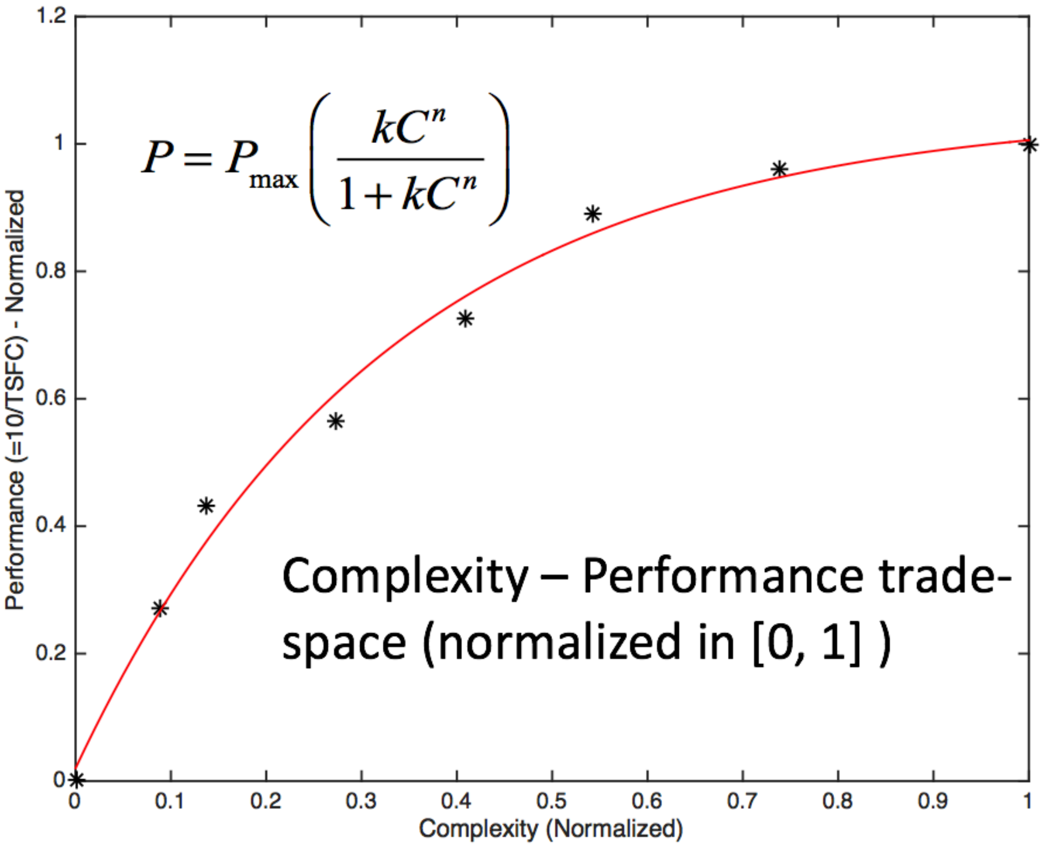
Complexity = 499

Complexity increase +42%

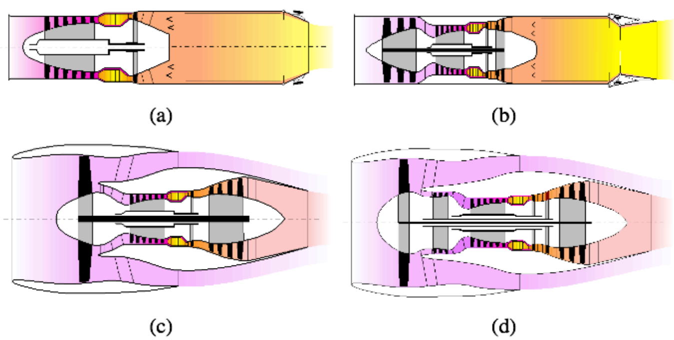
	C_1		C_2		C_3		C		C/C_{ML}		C_{new}/C_{old}
	Old	**New**	**Old**	**New**	**Old**	**New**	**Old**	**New**	**Old**	**New**	
Most Likely	161	188	126	184	1.51	1.69	351	499	1	1	1.42
Mean	179	244	141	240.4	1.51	1.69	392	650.3	1.12	1.30	1.65
Median	178	242	139	238.9	1.51	1.69	388	646.8	1.10	1.29	1.66
70 percentile	181	247.9	145	246.2	1.51	1.69	399.6	663.94	1.14	1.33	1.66

Trend towards more distributed architecture with higher structural complexity and significantly higher development cost*. Similar trend was observed in Printing Systems.

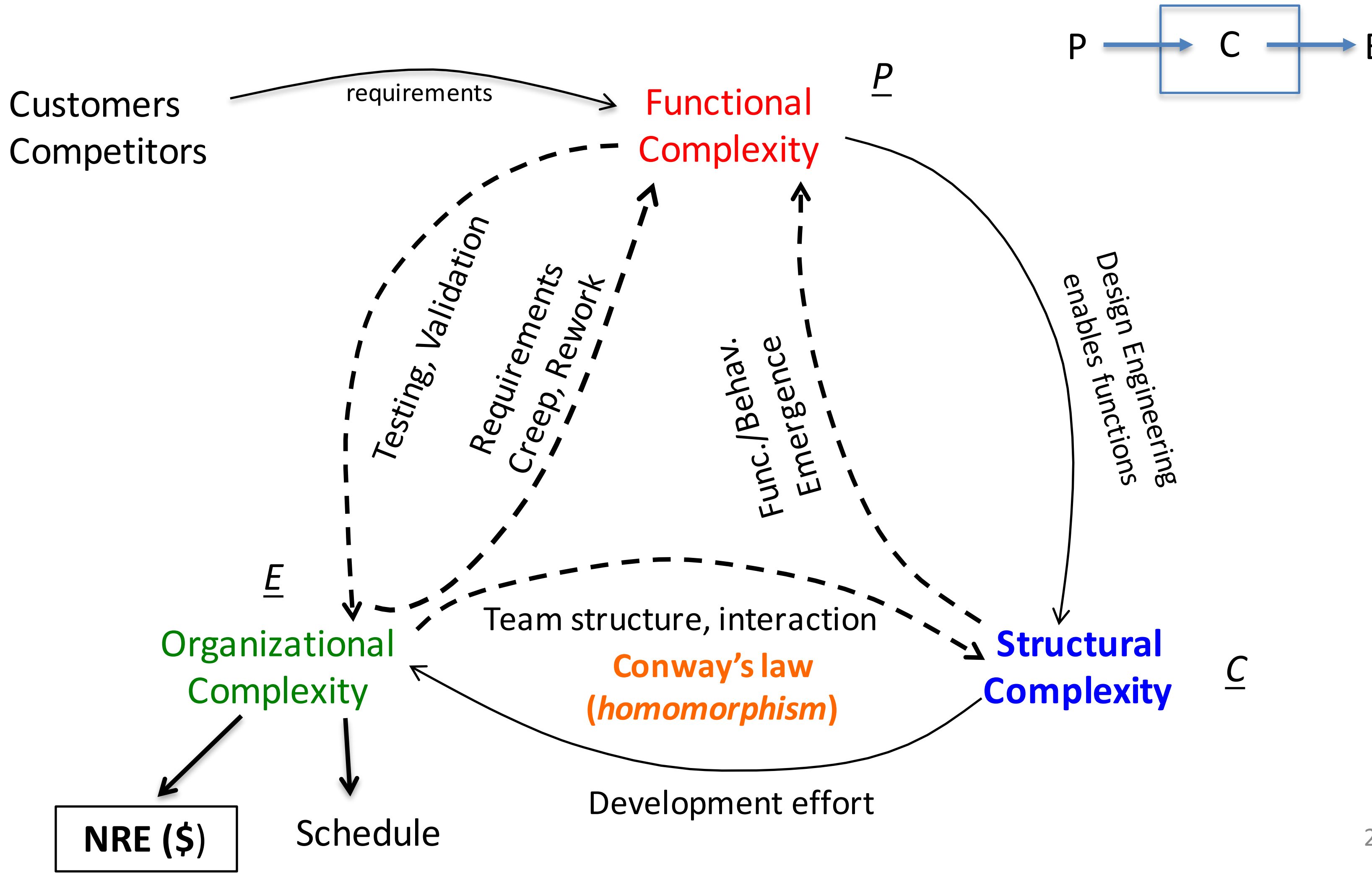
Diminishing Returns with Complexity



Left: Diminishing returns of normalized TSFC performance for air-breathing aircraft engines versus complexity, Bottom: evolution from turbojet to geared high BPR turbofans



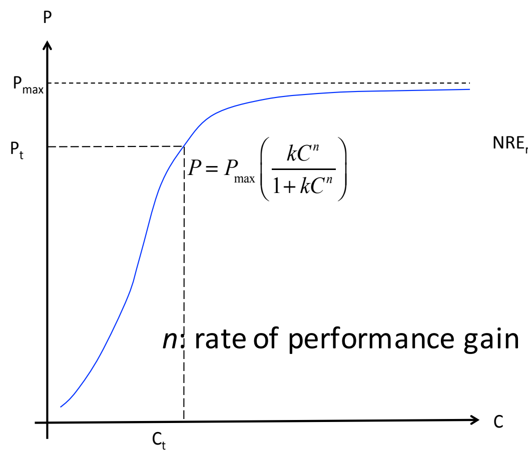
Three Dimensions of Complexity

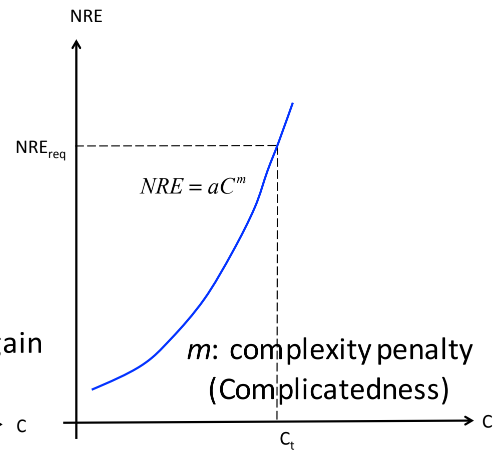


NRE Cost – Non-Recurrent Engineering Cost

Implication 1: Setting Complexity Targets

Complexity budget is the level of complexity that maximizes Value !



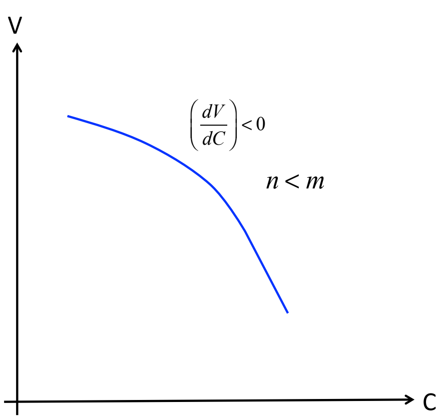


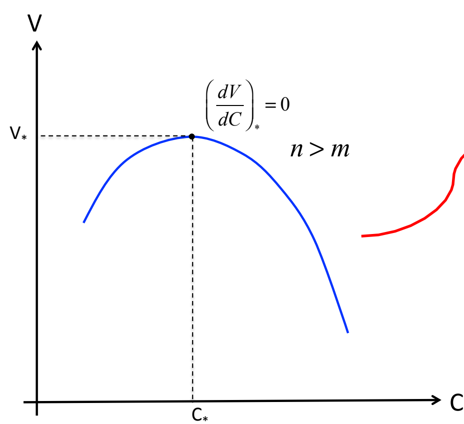
$$P = P_{max}\left(\frac{kC^n}{1+kC^n}\right)$$

$$NRE = aC^m$$

$$V = \frac{P}{NRE} = P_{max}\left(\frac{k}{a}\right)\left[\frac{C^{(n-m)}}{1+kC^n}\right] = S\left[\frac{C^{(n-m)}}{1+kC^n}\right]$$

Value V is the ratio of Performance P over non-recurring Effort E → what is V*?

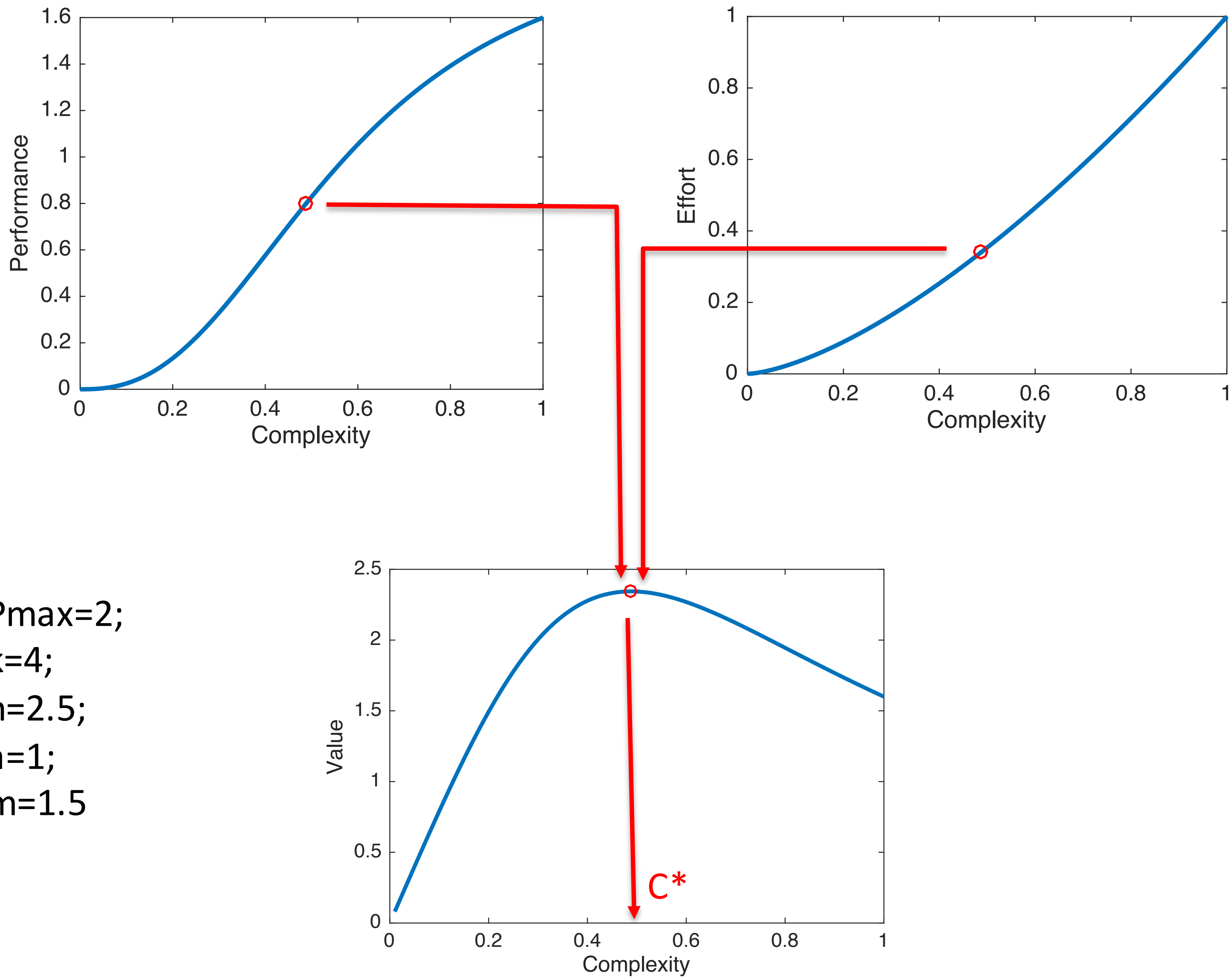




$$C_*^n = \frac{\left(\frac{n}{m}\right)-1}{k}; \quad P_* = P_{max}\left(1-\frac{m}{n}\right)$$

$$NRE_* = a\left[\frac{\left(\frac{n}{m}\right)-1}{k}\right]^{\frac{m}{n}}; \quad V_* = S\left(\frac{m}{n}\right)\left[\frac{\left(\frac{n}{m}\right)-1}{k}\right]^{\left(1-\frac{m}{n}\right)}$$

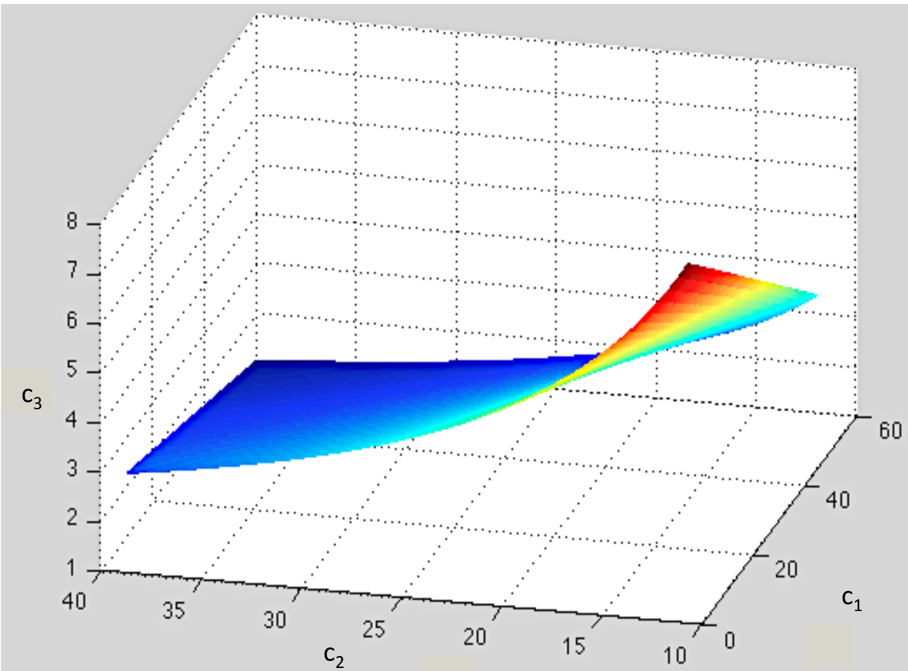
Example: Complexity Target to optimize Value



Pmax=2;
k=4;
n=2.5;
a=1;
m=1.5

Implication 2: Iso-Complexity Tradeoffs

- Once we define the level of complexity, there are different ways to distribute this total structural complexity, C into its three constituents $\{C_1, C_2, C_3\}$: *Iso-Complexity Surface*

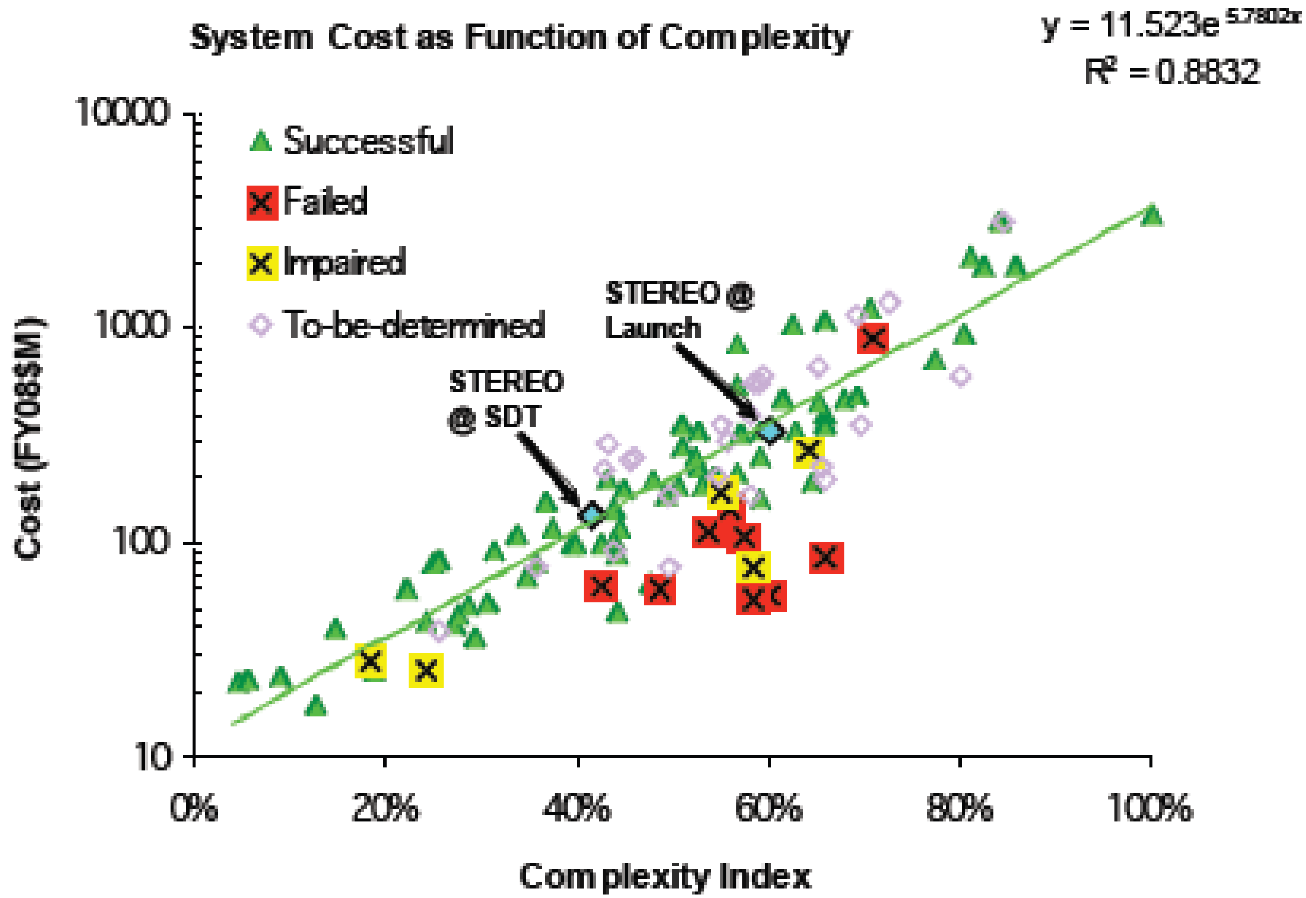


Iso-complexity surface: n = 20 components, assuming, c_1 in [10,60]; c_2 in [12,40] and C = 100.

- Tradeoff between (i) complex components and simple architecture, or (ii) simpler components and more complex architecture.
- Choice can be made depending on complexity handling capabilities of the development organization. E.g.
  - Excellent component designers
  - Skilled Systems integrators
  - Etc …

Implication 3: Minimum Development Effort (NRE)

- **CoBRA (Aerospace Corp., 2008) – Complexity Index based on analysis of historical data.**
- **Projects that were highly complex but tried to cut development cost had high failure rates**



The First Law of Systems Science: Conservation of Complexity

- First Law of Thermodynamics:
  - Conservation of Energy
  - The change in internal energy ΔU is equal to the heat *Q* added to the system minus the work *W* done by the system.

$$\Delta U = Q - W.$$

- The First Law of Systems Science:
  - Conservation of Complexity
  - The change in structural complexity C of the system is equal to a proportional change in expected performance P minus the change in effort E expended by the organization

$$\Delta C = \mu \Delta P - \varepsilon \Delta E$$

$$\varepsilon = -\frac{C^{1-m}}{2am} \qquad \mu = \frac{(1+kC^n)^2}{2Pmaxkn C^{n-1}(1-kC^n)}$$

Summary of key points

- **YES we have progressed in the last 20 years !**
- Structural complexity C of man-made systems has been increasing
- This is driven by customer needs and competition → functional performance P → structural complexity C → organizational effort E
- A rigorous measure of complexity is based on graph energy of DSM
  - C = C1+ C2*C3;
  - C3: Graph Energy is a measure of topological complexity
  - Iso-complexity based budgeting with clear targets is needed
- **First Law of Systems Engineering (according to de Weck-Sinha):**
  - **Conservation of Complexity**
  - **Given a set of functional requirements P, establish minimum needed structural complexity C, and calculate organizational effort E (NRE) to satisfy the first law**
- Violating the first law can lead to project or system failure !

Questions?

Comments?

Structural Complexity Metric

$$C = C_1 + C_2 . C_3$$

$$C(n,m,A) = \sum_{i=1}^{n} \alpha_i + \left(\sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} A_{ij} \right) \gamma E(A)$$

Normalization factor (1/n)

$$E(A) = \sum_{i=1}^{n} \sigma_i$$

Graph energy

Binary (0/1) conn. matrix

# of components

#interfaces

DSM

Components

Interfaces

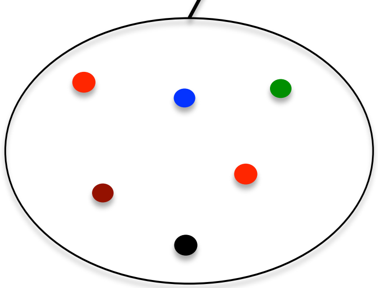
Architecture

Related to component engineering

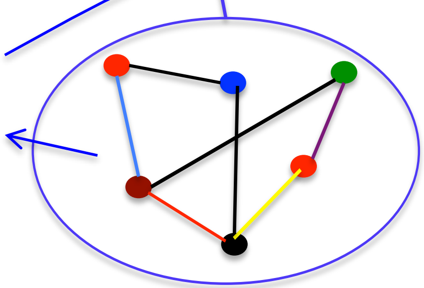
Related to interface design and mgmt.

Related to System Integration Effort

(topological complexity)



$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$



$$A = U \Sigma V^T$$

$$\Rightarrow A = \sum_{i=1}^{n} \sigma_i \underbrace{u_i v_i^T}_{R_i}$$

Metric Validity: Weyuker's Criteria

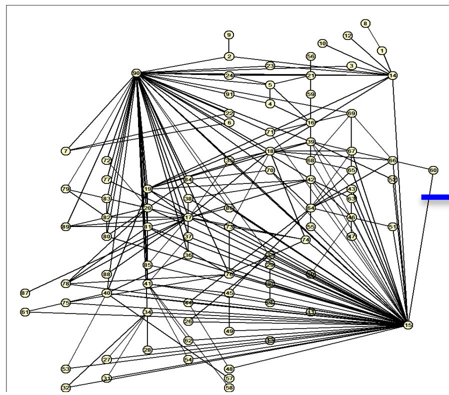
- Graph Energy stands out as both computable and satisfies Weyuker's criteria (1998) and establishes itself as a theoretically valid measure (i.e., construct validity) of complexity.

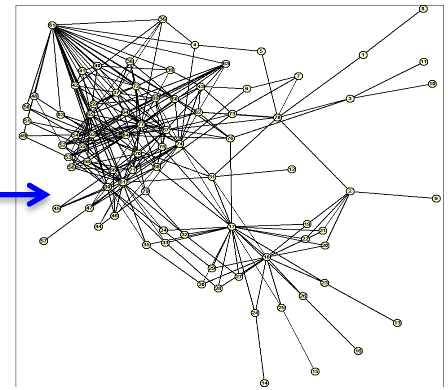
Complexity Measure	**Computability**	**Aspect emphasized**	**Weyuker's Criteria**
Number of components [Bralla, 1986]	✔	Component development (count-based measure)	✗
Number of interactions [Pahl and Beitz, 1996]	✔	Interface development (count-based measure)	✗
Whitney Index [Whitney *et al.*, 1999]	✔	Components and interface developments	✗
Number of loops, and their distribution []	✗	Feedback effects	✗
Nesting depth [Kerimeyer and Lindemann, 2011]	✗	Extent of hierarchy	✗
Graph Planarity [Kortler *et al.*, 2009]	✔	Information transfer efficiency	✗
CoBRA Complexity Index [Bearden, 2000]	✔	Empirical correlation in similar systems	✗
Automorphism-based Entropic Measures [Dehmer *et al.*, 2009]	✗	Heterogeneity of network structure, graph reconfigurability	✔
Matrix Energy / Graph Energy	✔	Graph Reconstructability	✔

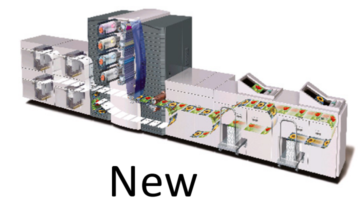
Example 1: Printing Engines

Old

Complexity = 186







New

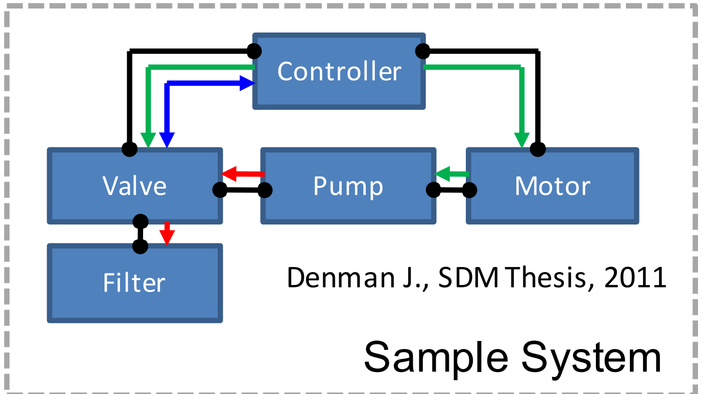
Complexity = 354

Complexity increase +90%

	C_1		C_2		C_3		C		C_{New}
	Old	**New**	**Old**	**New**	**Old**	**New**	**Old**	**New**	$/C_{Old}$
Most Likely	110.2	169	55.68	102.78	1.36	1.804	185.93	354.42	1.9062
Mean	125.62	213.6	63.29	130.6	1.36	1.804	211.69	449.2	2.122
Median	124.47	211.84	62.46	128.62	1.36	1.804	209.42	443.88	2.12
70 percentile	127	219	65.82	134.2	1.36	1.804	216.2	461.1	2.133

- Trend towards more distributed architecture with higher structural complexity and significantly higher development cost*

Example: Cyber-Physical System

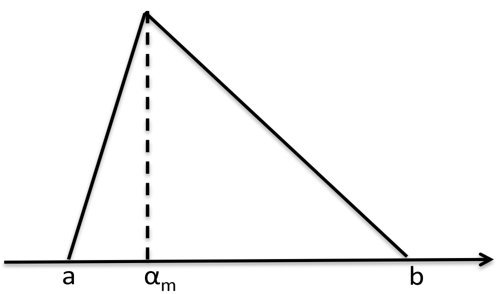


aggregation →

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Component	ID	Complexity
Controller	1	1.5
Pump	2	1.0
Valve	3	0.3
Filter	4	0.3
Motor	5	1.2

Comp. 1	Comp. 2	$1/c^{(k)}$
1	3	0.05
1	3	0.10
1	3	0.15
1	5	0.05
1	5	0.10
2	3	0.05
[illegible]	[illegible]	0.10
[illegible]	[illegible]	0.05
[illegible]	[illegible]	0.15
[illegible]	[illegible]	0.05
[illegible]	[illegible]	0.10



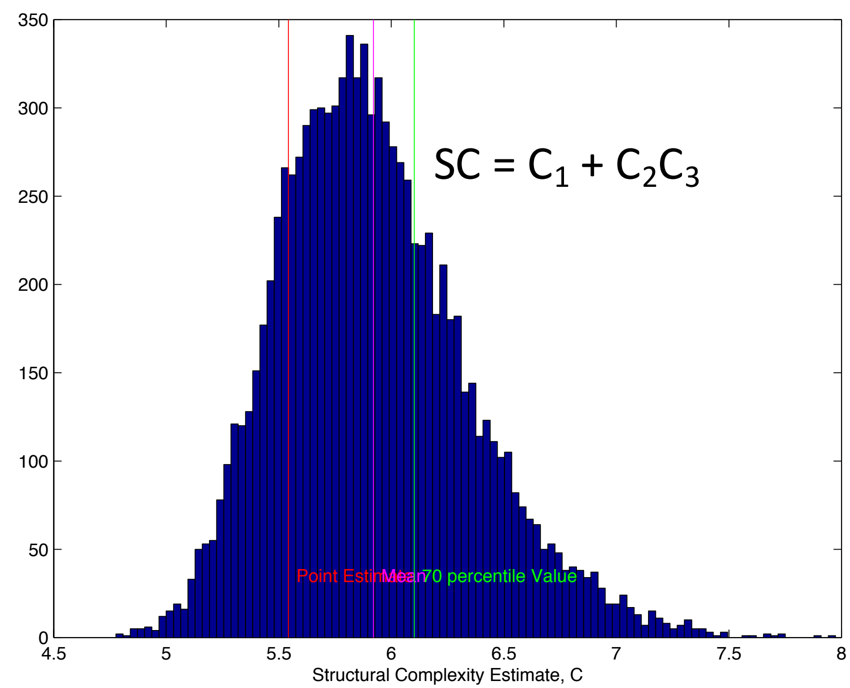
$(b-\alpha_m) = p(\alpha_m - a)$

$$p \in [1.0; 3.0]$$
$$a \in [0.8\alpha_m; 0.9\alpha_m]$$
$$b \in [1.1\alpha_m; 1.6\alpha_m]$$

$$\beta_{ij}^{(k)} = g(\alpha_i, \alpha_j, c^{(k)})$$

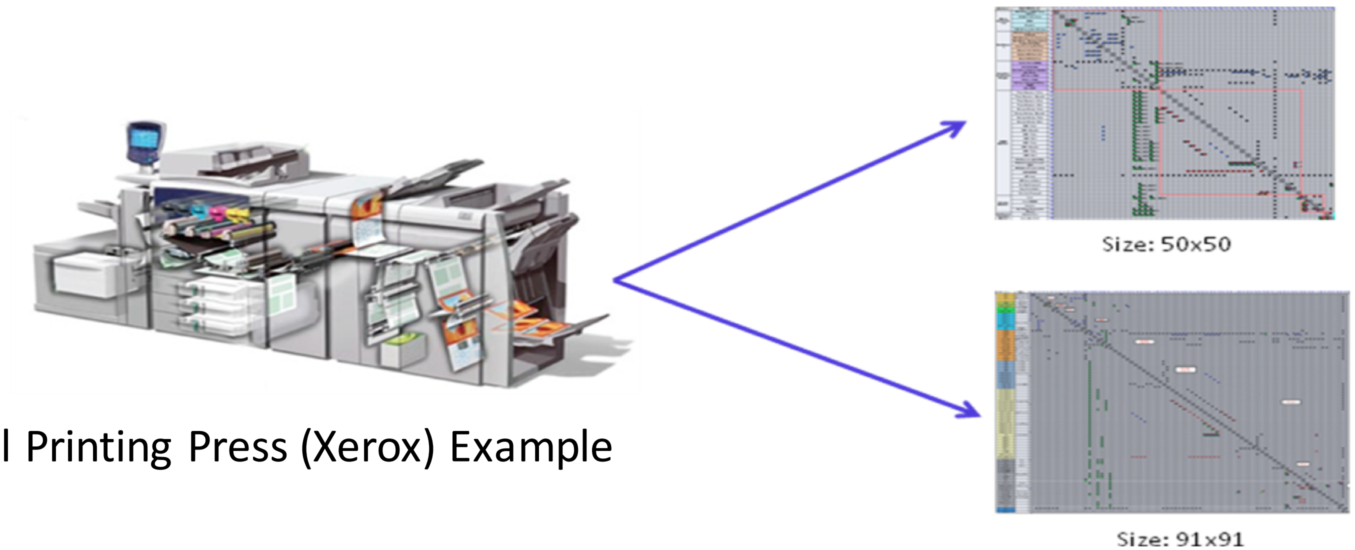
$$\beta_{ij}^{(k)} = \frac{\max(\alpha_i, \alpha_j)}{c^{(k)}},$$

$\forall \alpha_i, \alpha_j \neq 0$, *k is the interface type*



Structural Complexity Estimate, C

Complexity should be abstraction-Invariant



Digital Printing Press (Xerox) Example

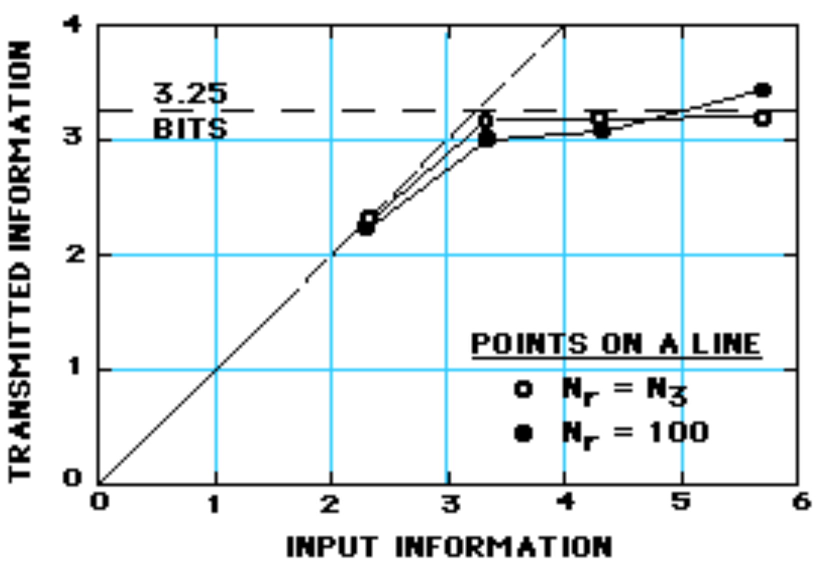
DSM attribute	Coarse Representation	Finer representation
System size, N	50	91
C_3	1.3534	1.3597

Functional Area	Coarse DSM (50x50)	Fine DSM (91x91)
ROS Assembly	4	10
Marking elements	16	38
Paper Path	7	12

Magic Number 7+/-2

- Human Cognitive Limits for Processing Information
- George Miller (1956)
- http://www.musanim.com/miller1956/

Position of a Pointer on Linear Interval



Auditory Pitch Experiments