

# Hidden Structures: Using Graph Theory to Explore Complex System of Systems Architectures

Matthew Potts<sup>1</sup>, Pia Sartor,<sup>1</sup> Angus Johnson<sup>2</sup> and Seth Bullock<sup>1</sup>

<sup>1</sup>University of Bristol

<sup>2</sup> Thales

Presented By: Matthew Potts

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Abstract

- Increasing interconnectivity in complex engineered systems of systems (SoS) leads to difficulties ensuring/understanding quality, flexibility, reliability, etc.
  - Reductionist approaches may fail to provide insight into key properties.
- Here, a NATO Architecture Framework for a Search and Rescue use case, is analysed using methods and concepts from graph theory:
  - E.g., degree distribution, density, connected components, modularity.
- This approach supports architectural decision making:
  - E.g., dependency allocation, boundary identification, areas of focus and selection between architectures.
- ...but it must be deployed with care.









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- Part 1 Introduction
  - Motivation
  - Current Approaches
  - Proposed Approach
- Part 2 Methodology
- Part 3 Analysis
- Part 4 Discussion
- Part 5 Conclusions and Further Work









## Motivation (1): SoS Design

- System of Systems (SoS) Characteristics:
  - Autonomy
  - Belonging
  - Connectivity
  - Diversity
  - Emergence

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 Emergence...the principle that entities exhibit properties which are only meaningful when attributed to the whole, not to its parts
[Boardman & Sauser]

#### INCOSE 2025 Vision







#### Motivation: System Architecture

				Behaviour					
	Taxonomy	Structure	Connectivity	Processes	States	Sequences	Information	Constraints	Roadmap
Concepts	C1 Capability Taxonomy NAV-2, NCV-2 AV-2, StV-2	C2 Enterprise Vision NCV-1 StV-1	Capability Dependencies NCV-4 StV-4	C4 Standard Processes NCV-6 StV-6	C5 Effects		C7 Performance Parameters NCV-1 StV-1	C8 Planning Assumptions	Cr Capability Roadmap <u>NCV-3</u> StV-3
Service Specifications	Service Taxonomy NAV-2, NSOV-1 AV-2, SOV-1		Sa Service Interfaces NSOV-2 SOV-2	Service Functions NSOV-3 SOV-5	S5 Service States NSOV-4b SOV-4b	Service Interactions NSOV-4c SOV-4c	S7 Service I/F Parameters NSOV-2 SOV-2	Sa Service Policy NSOV-4a SOV-4a	Sr Service Roadmap
Logical Specifications	L1 Node Types NAV-2 AV-2	L2 Logical Scenario NOV-2 OV-2	L3 Node Interactions NOV-2, NOV-3 OV-2, OV-3	L4 Logical Activities NOV-5 OV-5	L5 Logical States	L6 Logical Sequence NOV-6c OV-6c	L7 Logical Data Model NSV-11a OV-7	L8 Logical Constraints NOV-6a OV-6a	Lines of Development NPV-2 AcV-2
Physical Resource Specifications	P1 Resource Types NAV-2, N5V- 2a,7,9,12 AV-2, SV-2a,7,9,12	P2 Resource Structure NOV-4,NSV-1 OV-4, SV-1	P3 Resource Connectivity NSV-2, NSV-6 SV-2, SV-6	P4 Resource Functions NSV-4 SV-4	P5 Resource States NSV-10b SV-10b	P6 Resource Sequence NSV-10c SV-10c	P7 Physical Data Model NSV-11b SV-11	P8 Resource Constraints NSV-10a SV-10a	Pr Configuration Management NSV-8 SV-8
Deployed Resources	D1 Master Data NAV-2 AV-2	D2 Deployed Resources NOV-4 OV-4	NAFV4 Viewpoints					Dr Deployment Schedule NCV-5 StV-5	





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- "A design structure matrix (DSM) provides a simple, compact, and visual representation of a complex system that supports innovative solutions to decomposition and integration problems" [Browning]
- A Domain Mapping Matrix (DMM) include multiple domains
- The two can be combined to create a Multiple-Domain Matrix (MDM)



**Current Approaches: Matrices** 

DSM, DMM & MDM Examples taken from Design Society







#### **Current Approaches: Matrices**

	Environmenta Domain	Social Domain Functional Domain		Technical Domain	Process Domain		
	System Drivers	Stakeholders	Objectives	Functions	Objects	Activities	
System Drivers	System Drivers X System Drivers	Stakeholders X System Drivers	Objectives X System Drivers	Functions X System Drivers	Objects X System Drivers	Activities X System Drivers	
Stakeholders	System Drivers X Stakeholders	Stakeholders X Stakeholders	Objectives X Stakeholders	Functions X Stakeholders	Objects X Stakeholders	Activities X Stakeholders	
Objectives	System Drivers X Objectives	Stakeholders X Objectives	Objectives X Objectives	Functions X Objectives	Objects X Objectives	Activities X Objectives	
Functions	System Drivers X Functions	Stakeholders X Functions	Objectives X Functions	Functions X Functions	Objects X Functions	Activities X Functions	
Objects	System Drivers X Objects	Stakeholders X Objects	Objectives X Objects	Functions X Objects	Objects X Objects	Activities X Objects	
Activities	System Drivers X Activities	Stakeholders X Activities	Objectives X Activities	Functions X Activities	Objects X Activities	Activities X Activities	

Engineering Systems Matrices taken from Bartolomei et al 2012

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#### **Current Approaches: Graph Theory**



Structural Complexity Metric taken from Sinha & De Weck 2013









#### **Current Approaches: Graph Theory**

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Manufacturing System Graph Theoretic Network taken from Harrison 2016









#### **Current Approaches: Graph Theory**



Developing a Higraph from a tree taken from Aboutaleb 2017









- Analyse to explore the underlying structure and relationships of a complex SoS architecture
- Support shared understanding
- Support decision making
- Coherent with SoS engineering methodologies and system architecture frameworks







### Methodology

- Part 1 Introduction
- Part 2 Methodology
  - Problem Characterisation
  - Graph Based Model
- Part 3 Analysis
- Part 4 Discussion
- Part 5 Conclusions & Further Work



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#### **Problem Characterisation**



OV1a High Level Operational Concept Graphic View taken from NAFv4 Chapter 2 Example,. Libert & Garnier J-L

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#### **Problem Characterisation**

System	Complex System	System of Systems	SAR Use Case Characterisation
Interacting elements [SEBoK, ISO 15288, Checkland, Boardman & Sauser][3-5]	Interconnected components [Bar-Yam, Mitchell, Waldrop]	Interconnected (connectivity) and diverse components or systems [Maier, Boardman & Sauser]	Heterogeneous components and relationships between them
Deliberately brought together [SEBoK, ISO 15288, Checkland, Boardman & Sauser]	Behaviour cannot be inferred from component behaviour [Bar-Yam, Mitchell, Waldrop, Crawley et al]	Constituent systems deliberately brought together (belonging) [Maier, Boardman & Sauser, ISO 15288]	Although aspects of SAR could be conducted by individual systems, a full maritime SAR capability relies on the overall SoS availability.
Defined purpose [ISO 15288]	May be adaptive and self organised (lack of centralised control) [Kaisler & Madey, Miller, Ladyman et al]	Defined purpose [Maier, Jamshidi]	The SAR SoS serves a clear purpose of Search and Rescue in the maritime domain.
May exhibit emergence [Checkland, Boardman & Sauser]	Capacity for emergence [Bar-Yam, Mitchell, Waldrop, Crawley et al]	Operational & managerial independence (including autonomy) [Maier, Jamshidi, Cocks]	Each constituent system is a system in their own right with their own management and operations, brought together for a common purpose.
		Capacity for emergence [Maierer, Boardman & Sauser, Jamshidi, ISO 15288]	Emergent behaviour is the ability of the SoS to effectively coordinate a search operation over a large area for a small, moving object and recover it with precision and speed to a place of safety.

Table of key characteristics for Systems, Complex Systems, System of Systems & the SAR Use Case







#### **Problem Characterisation**







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#### **Graph Based Model**





#### **Graph Based Model**



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- Part 1 Introduction
- Part 2 Methodology
- Part 3 Analysis
  - Degree Distribution
  - Density
  - Strongly Connected Components
  - Community Detection (Modularity)
- Part 4 Discussion
- Part 5 Conclusions & Further Work









#### **Degree Distribution**

- Examining the in and out degree distribution highlights potentially important vertices:
  - High in degree and low Mout degree of interest loss or removal of that Mouties could have significant consequences.









Density

• How densely connected is the graph?

• 
$$D = \frac{|E|}{|V|(|V|-1)}$$
 (1)

- where D is the graph density, E is the number of edges and V is the number of vertices in the graph
- Hints at integration or dependency management challenge
- Potential to compare architectures









- Strongly connected if path (distinct sequence of edges connecting vertices) in each direction between each pair of vertices
- Groupings of vertices that can reach other: 'core' and 'periphery'



Strongly connected components of SAR directed graph

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Rescue Service

- Communities of vertices with many edges joining vertices of the same community and comparatively few between communities
- Alternative approach to partitioning











- Part 1 Introduction
- Part 2 Methodology
- Part 3 Analysis
- Part 4 Discussion
  - Utilisation
  - Limitations & Cautions
- Part 5 Conclusions & Further Work









- Enable visualisation of structure and relationships that may remain hidden
- Identify areas to focus attention on
- Alternative approach to partitioning, boundary and dependency allocation
- Metrics in isolation meaningless require organisational learning







**Limitations & Cautions** 

- Architecture: Art & Science
- Large abstraction heterogeneous entities and relationships modelled
- Selection of scope for graph based model
- Edge weights could distinguish importance/intensity of connection
- Only explored metrics in one 'timeframe'
- Metrics in isolation meaningless require organisational learning
  - Recommendation use for specific purpose; visualise structure, identify areas of interest, temper decisions







#### **Conclusions and Further Work**

- Part 1 Introduction
- Part 2 Methodology
- Part 3 Analysis
- Part 4 Discussion
- Part 5 Conclusions & Further Work
  - Further Work
  - Summary
  - Conclusions









#### **Further Work**

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- Include edge weights
- Dynamic considerations of relationships
- Additional Use Cases



Tactical Military Communications Enterprise Architecture Directed Graph







- Challenge: supporting decision making on complex SoS architectures
- Graph based models have been widely used to identify important entities
- Graph based models can be used to explore complex SoS architectures
- Topological properties such as degree distributions, density, strongly connected components and community identification can support investment decisions, boundary selection, a new perspective on partitioning
- Such an approach has several cautions
- Recommended to be used as an additional perspective







Conclusions

- Complimentary view
- Integrate data already available into a single directed graph for analysis
- Potentially important characteristics become tractable
- May support decision making around complex SoS architectures
- Further work needed to qualify the utility of this approach







# Thank you

Any Questions?





