Performance Analysis of the Matched-Pulse-Based Fault Detection

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Outline

- Introduction
- The Matched Pulse Approach
- Topological Analysis
- Mathematical analysis
    - Detection Gain
    - Simulation results
    - Signal to Noise Ratio (SNR)
- Experimental results
- Conclusion and perspectives

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Introduction

200 Km

4 Km

40 – 400 Km

Faulty electrical wiring

↓

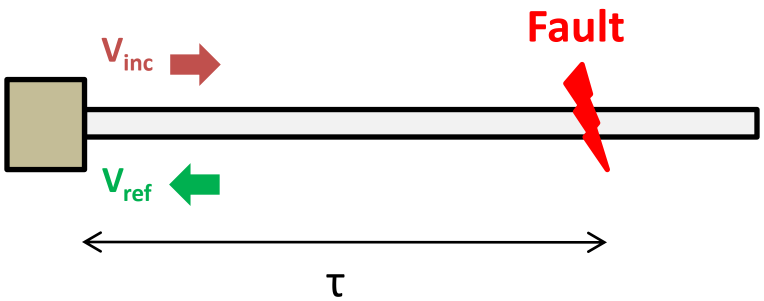
Problems of security, maintenance cost, etc.

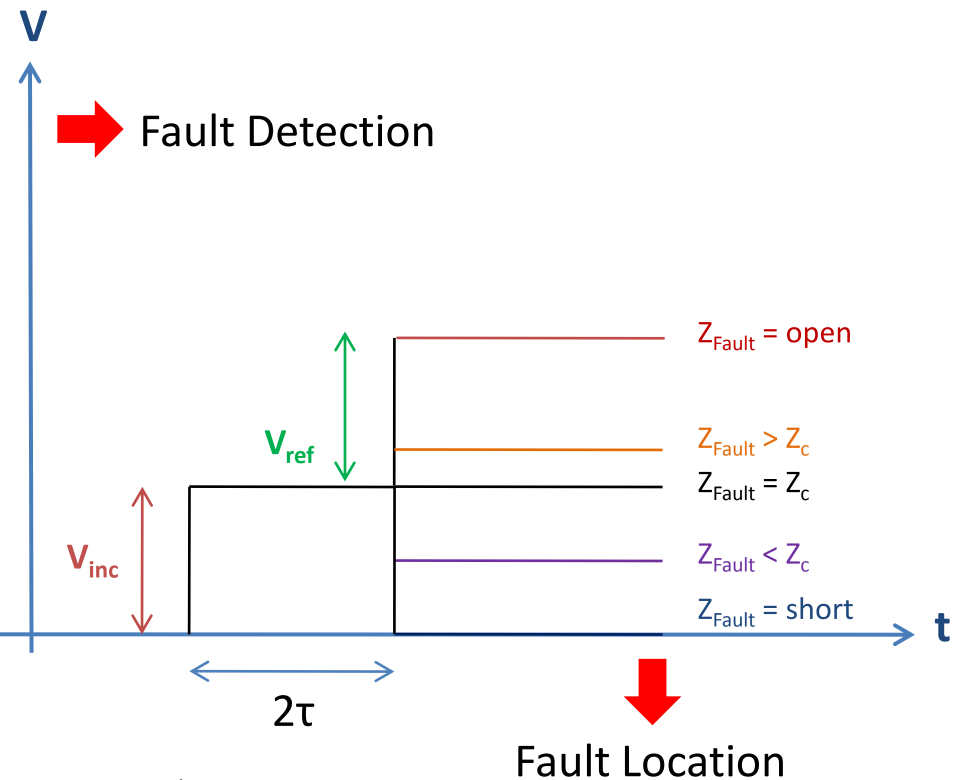
2500 Km

1200 Km

Existing wire testing methods

Reflectometry methods:





Limitations:

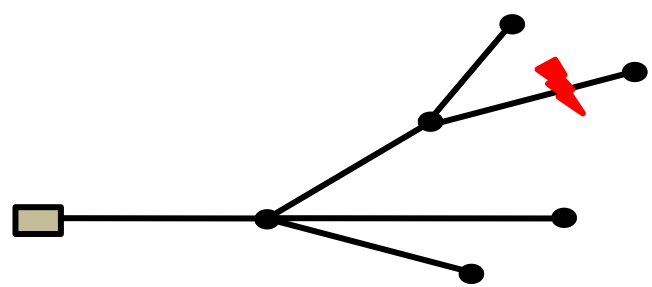
Faults → **Hard** (generally detectable)

→ **Soft** (difficult to detect)

Complex wire networks

Faults can be masked by discontinuities

Reflections disperse the energy of the fault's echo



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The Matched Pulse (MP) Approach

radar

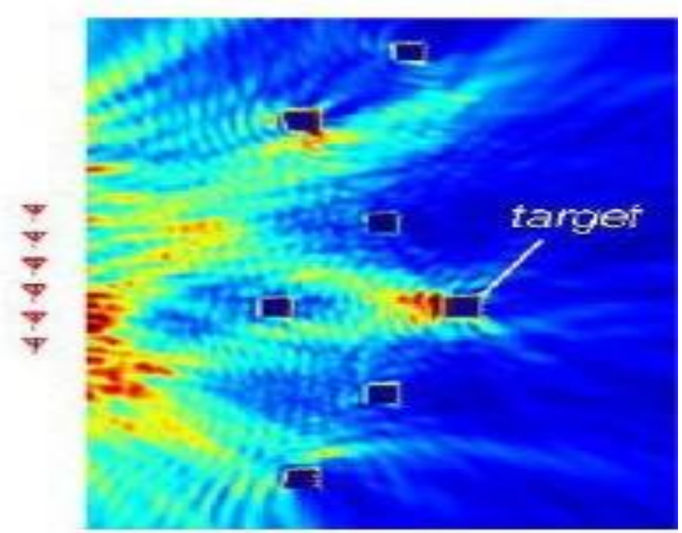
Wire networks

A target to be detected in a cluttered environment

A soft fault to be detected in a complex network

Solution : use the properties of **Time Reversal** to synthesize a signal, **adapted** to the propagation channel

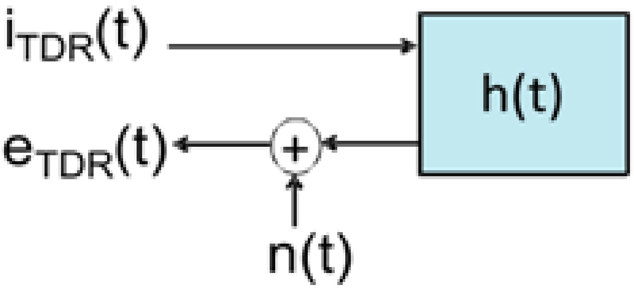
It has been demonstrated that such an adapted signal can minimize the clutter effect and maximize the target detectability.



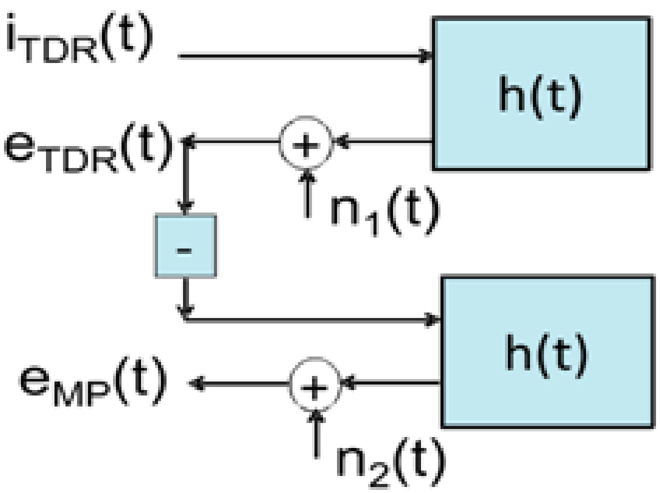
'Antenna array detection in highly cluttered environment using time reversal method' Y. Jiang, D. Stancil, and J.-G. Zhu

The Matched Pulse (MP) Approach

Standard TDR



MP



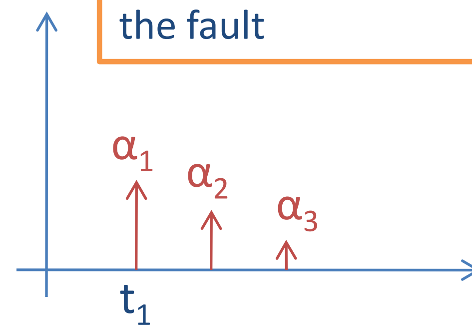
Example :

α_i : amplitude of the i-th peak

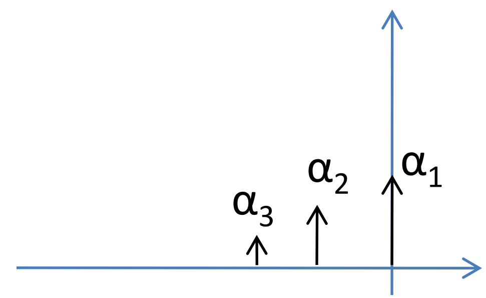
t_1: round trip time to the fault

1. We inject a pulse

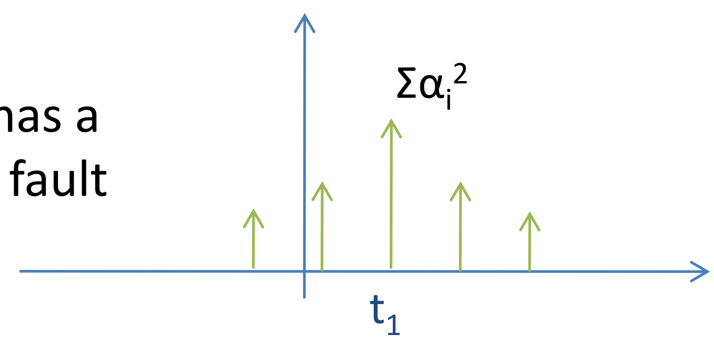
2. We suppose the impulse response has this form :



3. We time reverse and shift the obtained TDR echo :



4. The MP echo has a maximum at the fault position



Analysis Procedure

1. Topological analysis : study the impact of the network topology on the performance of the TDR

2. Mathematical analysis : to compare standard TDR and MP

   - Detection gain
   - Simulation results
   - Signal to Noise Ratio and detection probabilities
   - Discussion

3. Experimental results

In our analysis, we use the **difference system**, so the echoes of the soft faults are more easily observed.

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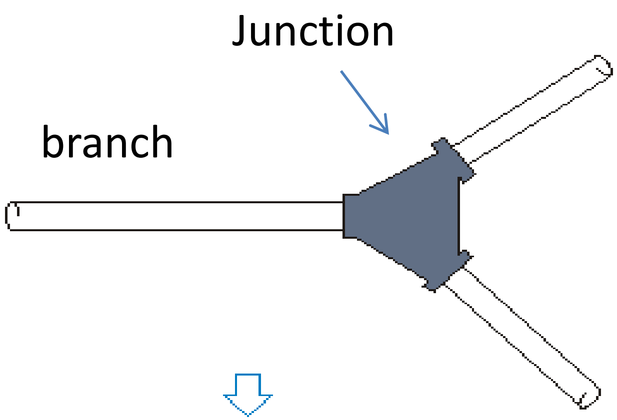
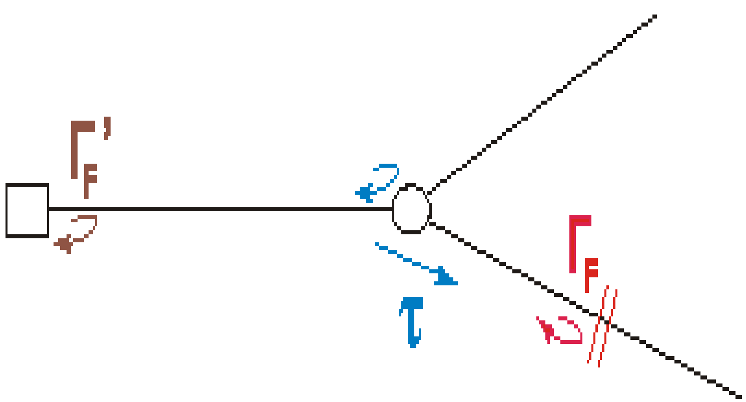
Topological Analysis





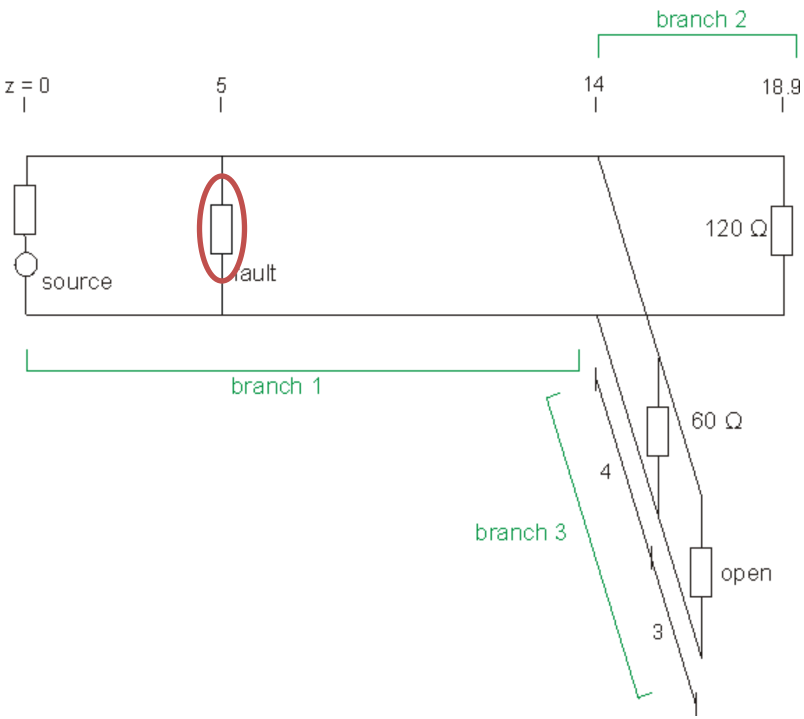
Shortest trajectory from the source to the fault :

- If we inject a pulse, its amplitude will be attenuated of $\tau^2\Gamma_F$ when back to the source

- A hard fault is not necessarily easy to detect

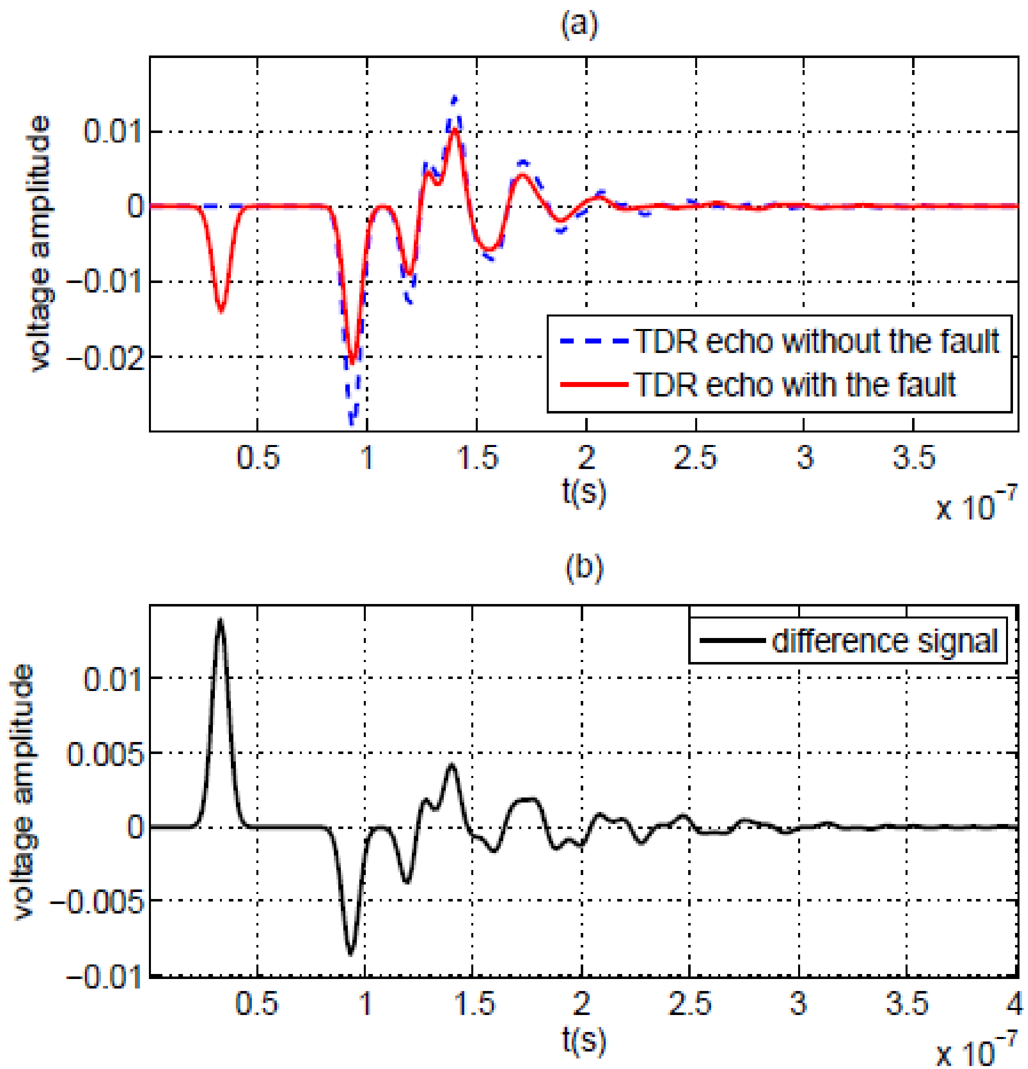
- Detectability depends on Γ'_F not on Γ_F

Topological Analysis



Fault: 200 Ω
Characteristic Impedance: 75 Ω

- The difference echo does not contain the peaks appearing before the fault.
- The first peaks are well resolved.



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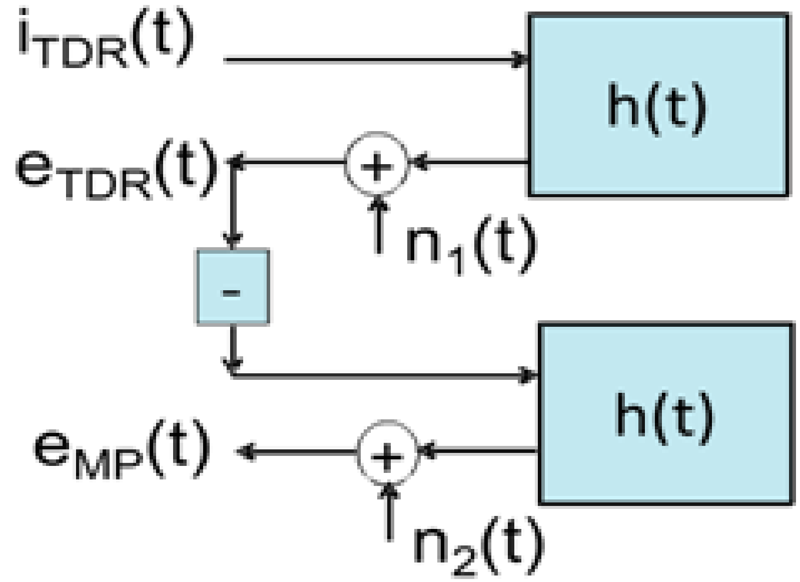
Mathematical Analysis: Detection Gain

α_i : amplitude of the i-th peak
t_i : temporal position of the i-th peak
i(t) : injected signal (energy normalized)

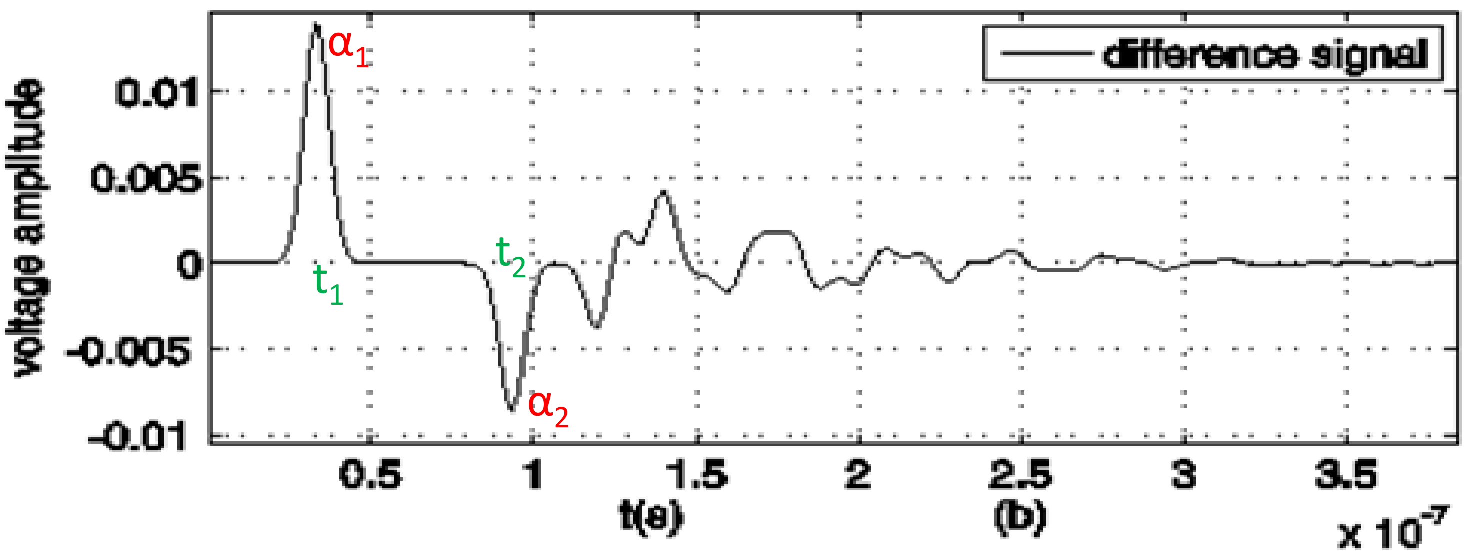


Impulse response of the system:

$$h(t) = \sum_i \alpha_i \delta(t - t_i)$$

Reflected signal :

$$e(t) = i(t) * h(t) = \sum_i \alpha_i i(t - t_i)$$



Mathematical Analysis: Detection Gain

$$G = \frac{\mathcal{E}^F_{\mathrm{MP}}/\mathcal{E}_{\mathrm{MP}}}{\mathcal{E}^F_{\mathrm{TDR}}/\mathcal{E}_{\mathrm{TDR}}}$$

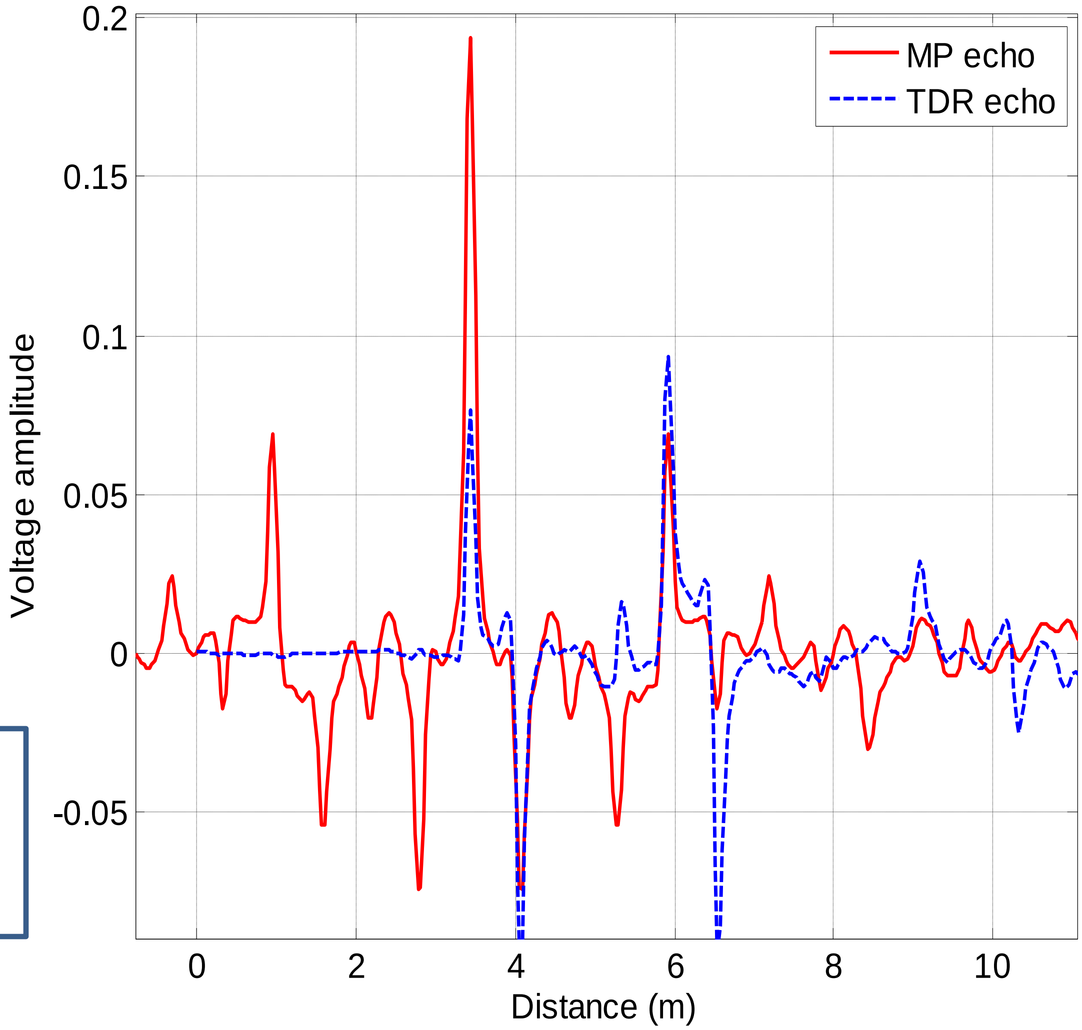
First peak in the TDR echo

$$\mathcal{E}^F_{\mathrm{TDR}} = \alpha_1^2$$

In the MP echo:

$$\mathcal{E}^F_{\mathrm{MP}} = \left|\sum_i \alpha_i^2\right|^2$$

$$G = \frac{\sum_{i=1}^{\infty} \alpha_i^2}{\alpha_1^2} \geq \frac{\sum_{i=1}^{N} \alpha_i^2}{\alpha_1^2}$$



Mathematical Analysis: Detection Gain

$$G = \frac{\sum_{i=1}^{\infty} \alpha_i^2}{\alpha_1^2}$$

- The formula allows us to predict the MP performance based on the system's topology
- The more echoes we have, the better the gain is
- The network's complexity increases the number of echoes, thus increasing the gain.

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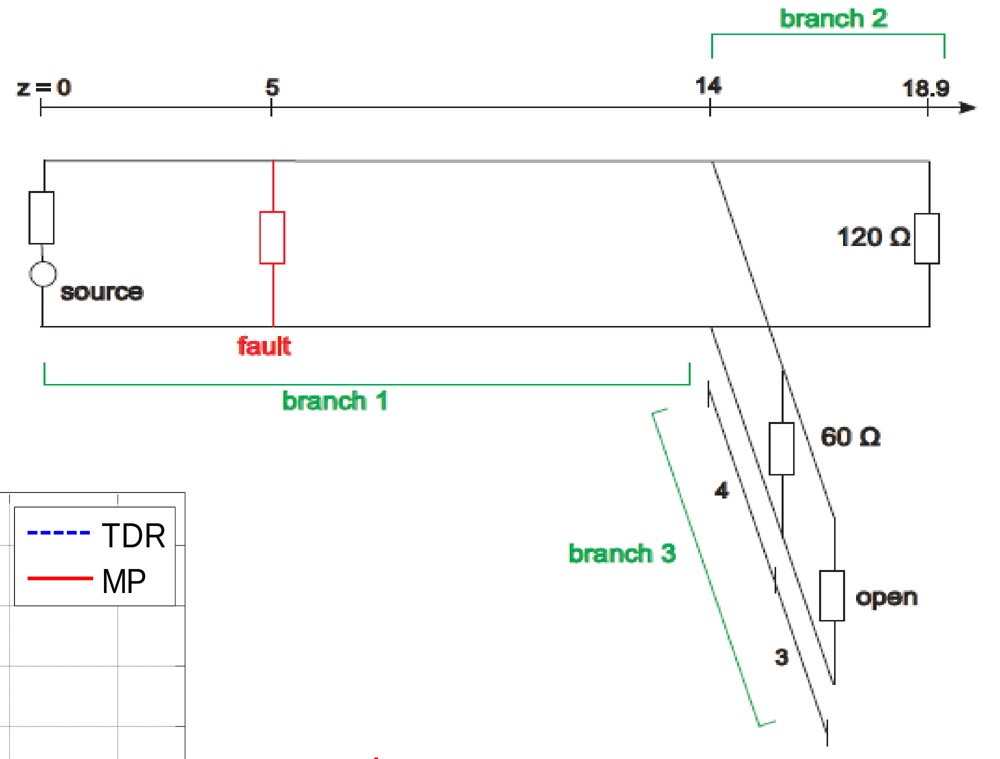
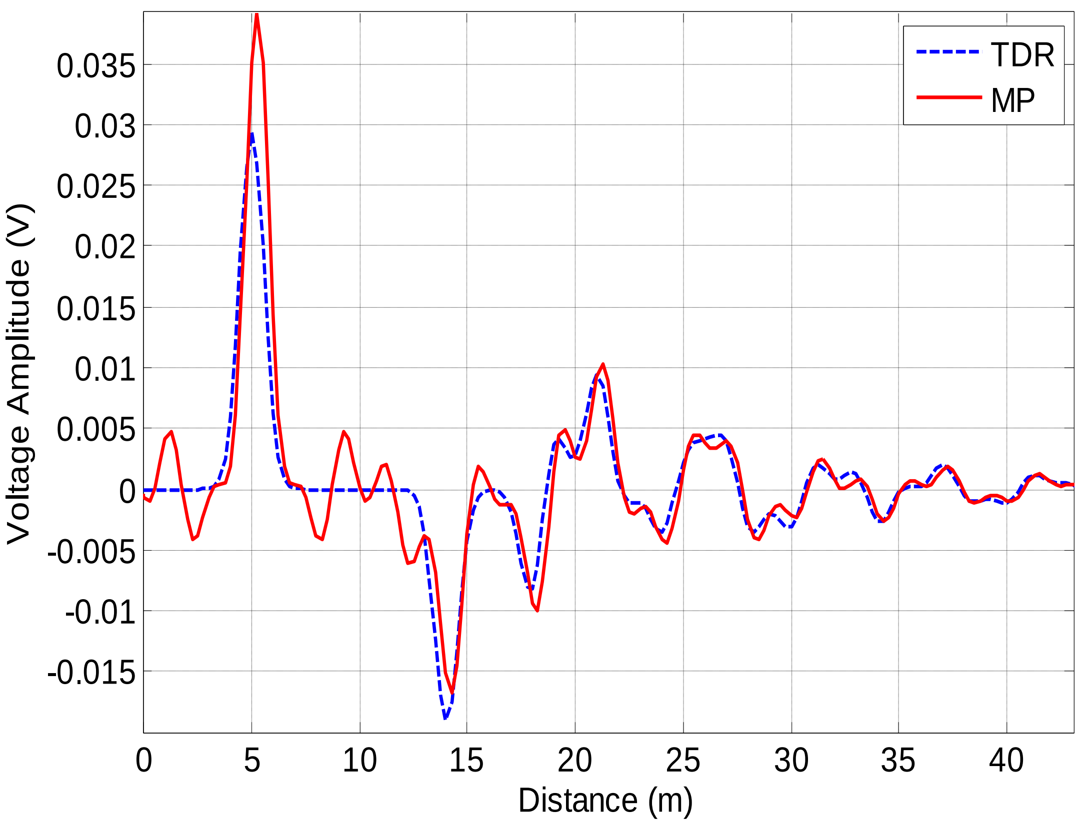
Simulation results

Fault value (Ω)	$\|\Gamma_F\|$	Predicted gain (lower bound)	Calculated gain
Short	1	1.16	1.18
30	0.56	1.34	1.38
150	0.2	1.53	1.61
600	0.06	1.61	1.72





Fault = 600 Ω

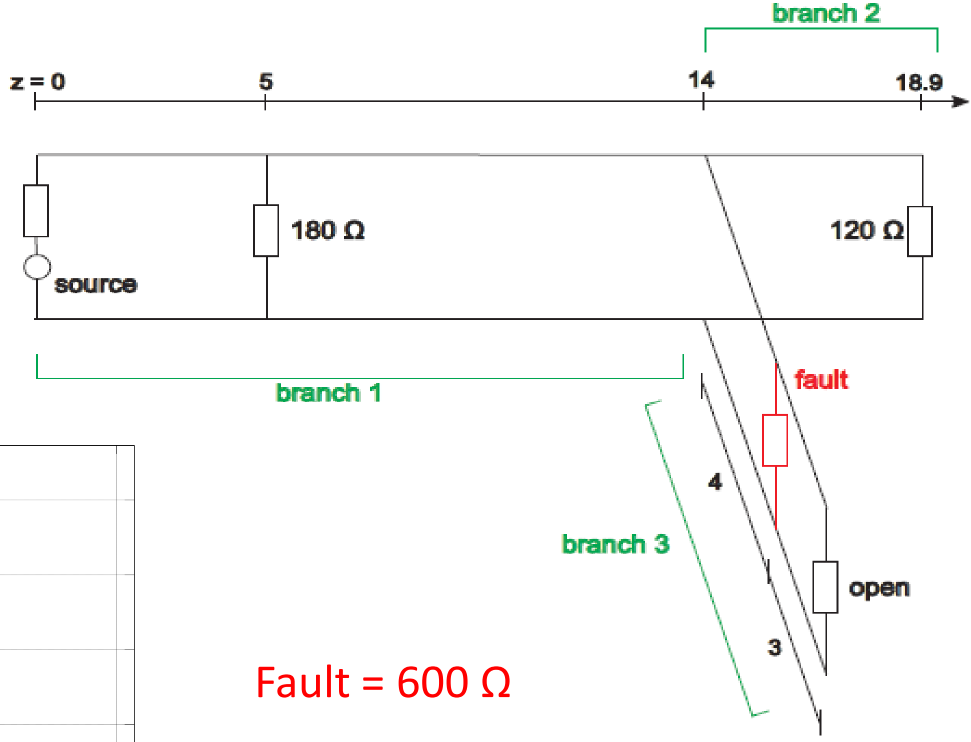
MP fault peak / TDR fault peak = **1.33**

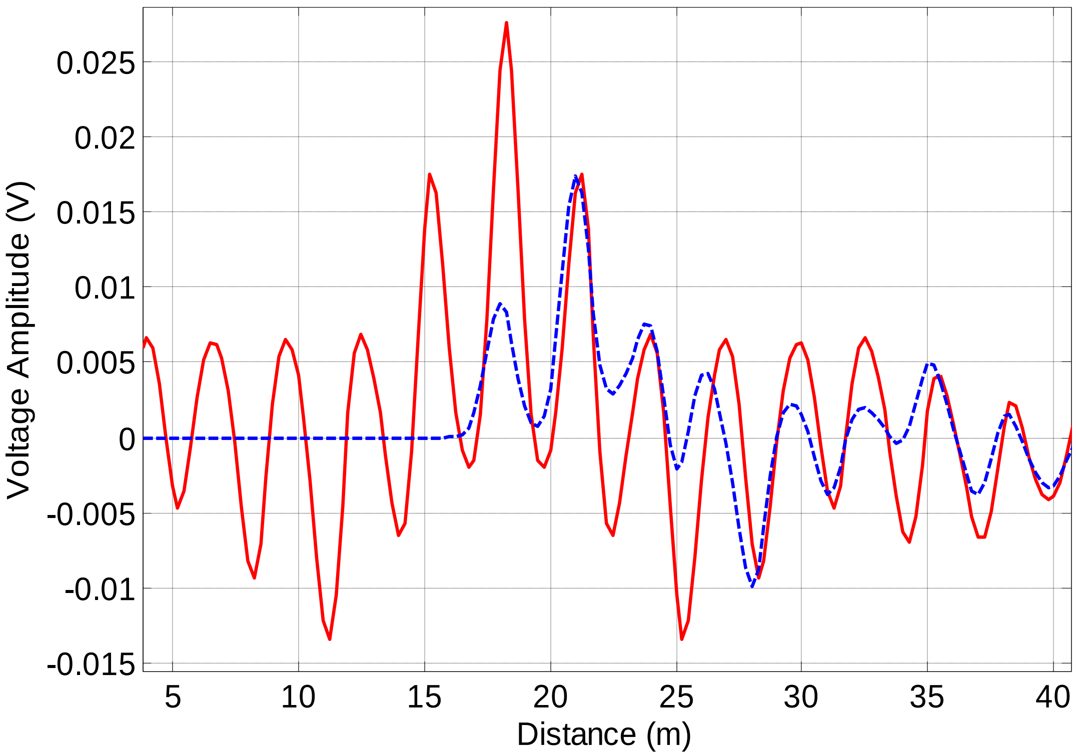
MP contrast = **8.36**

TDR contrast = **1.55**

Simulation results

Fault value (Ω)	$\lvert\Gamma_F\rvert$	Predicted gain (lower bound)	Calculated gain
Short	1	2.44	3.22
30	0.56	3.68	4.38
150	0.2	4.95	6.42
600	0.06	5.75	8.51





Fault = 600 Ω

MP fault peak / TDR fault peak = **2.7**

MP contrast = **1.58**

TDR contrast = **0.51**

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Mathematical Analysis: SNR

Assumptions :
- Additive white Gaussian noise
- t = 0 : recording instant

SNR

TDR case :

$$SNR|_{TDR} = \frac{\alpha_1^2}{2N_0}$$

MP case :

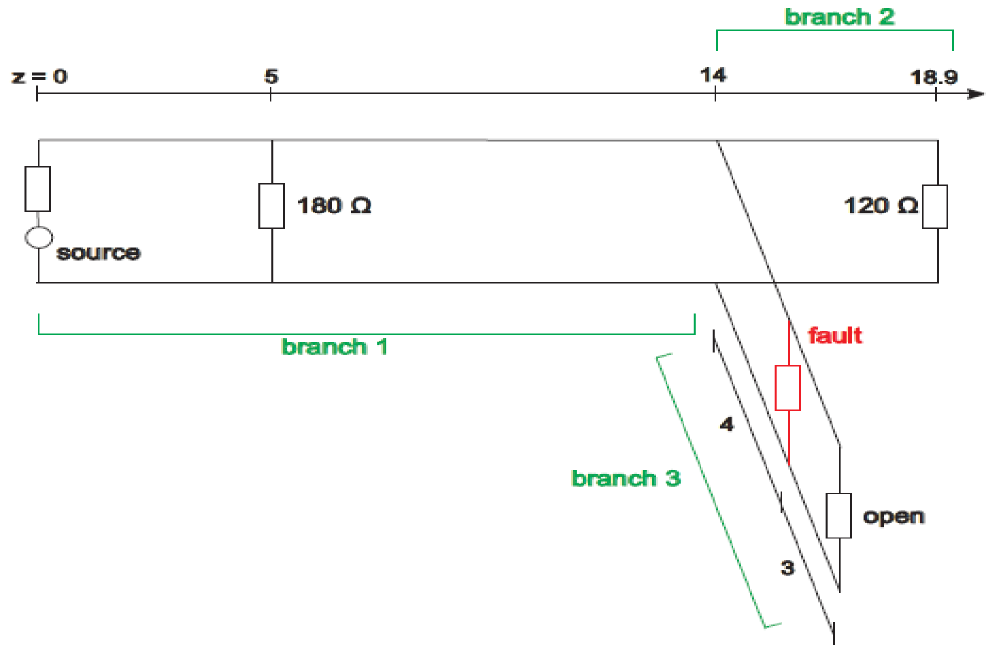
$$SNR|_{MP} = \frac{\sum \alpha_i^2}{2N_0(1 + \sum \alpha_i^2)}$$

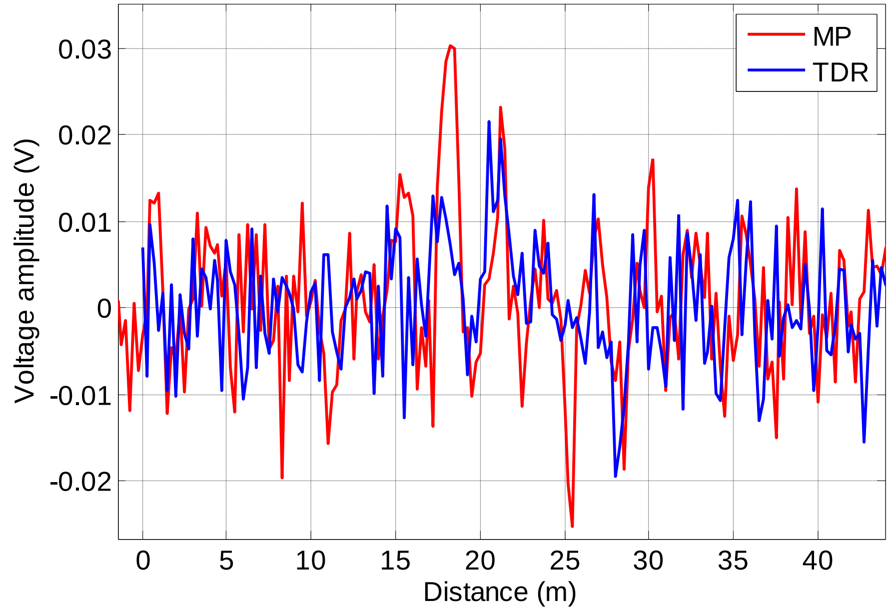
$$R = \frac{SNR|_{MP}}{SNR|_{TDR}}$$

$$R = G\frac{1}{1 + \sum_i \alpha_i^2}$$

Worst case :
R = G/2

If we want
R>1 ⇨ G>2



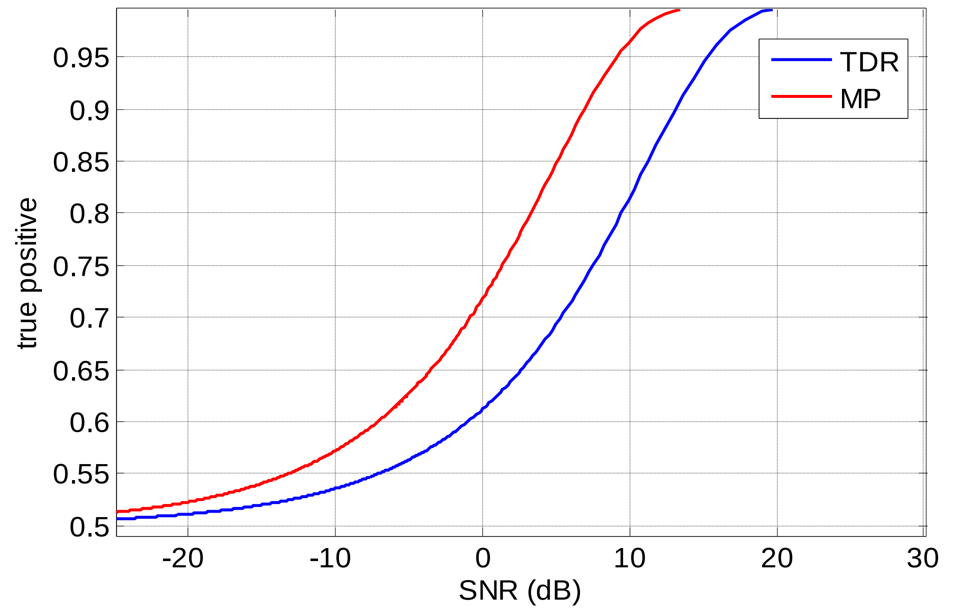


Mathematical Analysis: probabilities

Detection probability

$$p_{TDR}^{TP} = \frac{1}{2} erfc\left(\frac{a_{th} - \alpha_1}{\sqrt{2N_0}}\right)$$

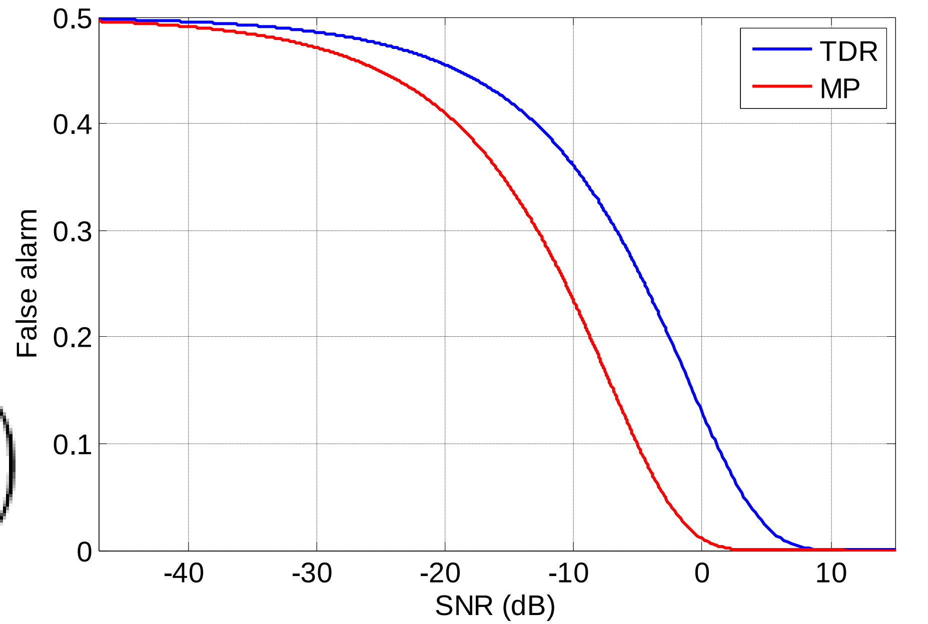
$$p_{MP}^{TP} = \frac{1}{2} erfc\left(\frac{a_{th} - \sqrt{\sum_i \alpha_i^2}}{2\sqrt{N_0(1+\sum_i \alpha_i^2)}}\right)$$



False Alarm

$$p_{TDR}^{FA} = \frac{1}{2} erfc\left(\frac{a_{th}}{\sqrt{2N_0}}\right)$$

$$p_{MP}^{FA} = \frac{1}{2} erfc\left(\frac{a_{th}}{2\sqrt{N_0(1+\sum_i \alpha_i^2)}}\right)$$



Discussion

2 systems

(A) **TDR**

(B) **MP**

$$N_B > N_A$$

We consider :

$$(a_{th})_A = \rho\alpha_1$$

$$(a_{th})_B = \rho\sqrt{\sum_i \alpha_i^2}$$

Same detection probability

$$\frac{(a_{th})_A - \alpha_1}{\sqrt{2N_A}} = \frac{(a_{th})_B - \sqrt{\sum_i \alpha_i^2}}{2\sqrt{N_B(1+\sum_i \alpha_i^2)}}$$

$$SNR|^A_{TDR} = SNR|^B_{TDR}\frac{G}{2}\frac{1}{1+\sum_i \alpha_i^2}$$

$$\sum_i \alpha_i^2 \ll 1$$

$$SNR|^A_{TDR} \approx SNR|^B_{TDR}\frac{G}{2}$$

In the system (B), for the same detection probability, the SNR obtained with the MP method is better of that obtained for the TDR method

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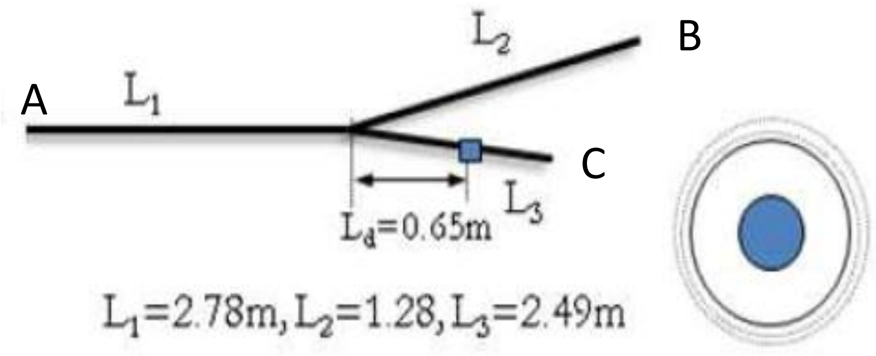
Experimental results

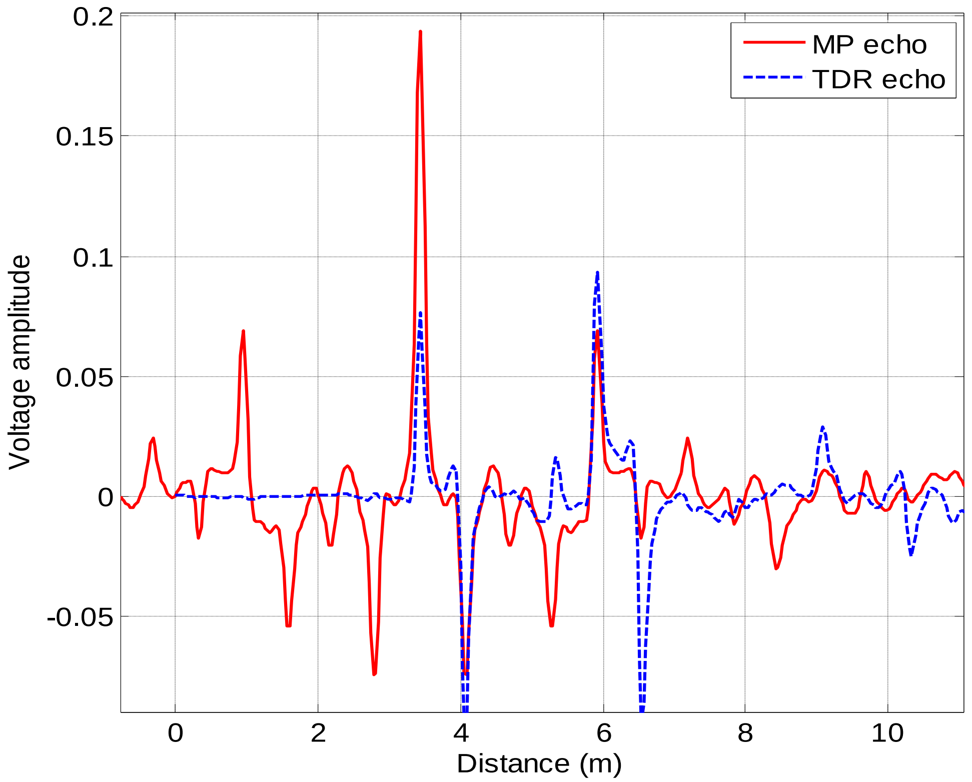
Studied network 1



A : injection point, impedance = 50 Ω
Injected pulse of amplitude 0.26 V.

B and **C** : terminations (open circuits)
Coaxial cables
Characteristic impedance : 60 Ω
Velocity of propagation : 0.826 c
Fault : dielectric removal on a distance of 3 cm

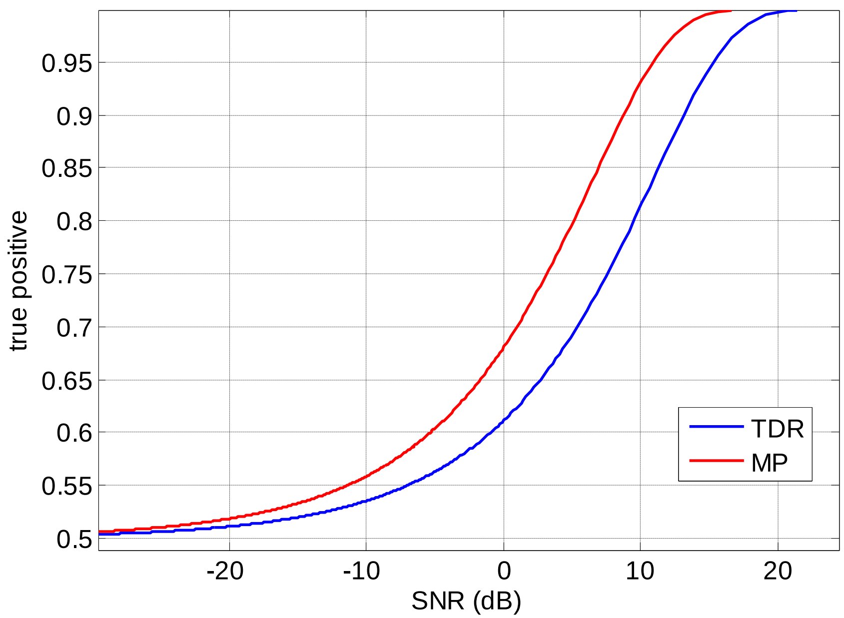
TDR and MP echoes



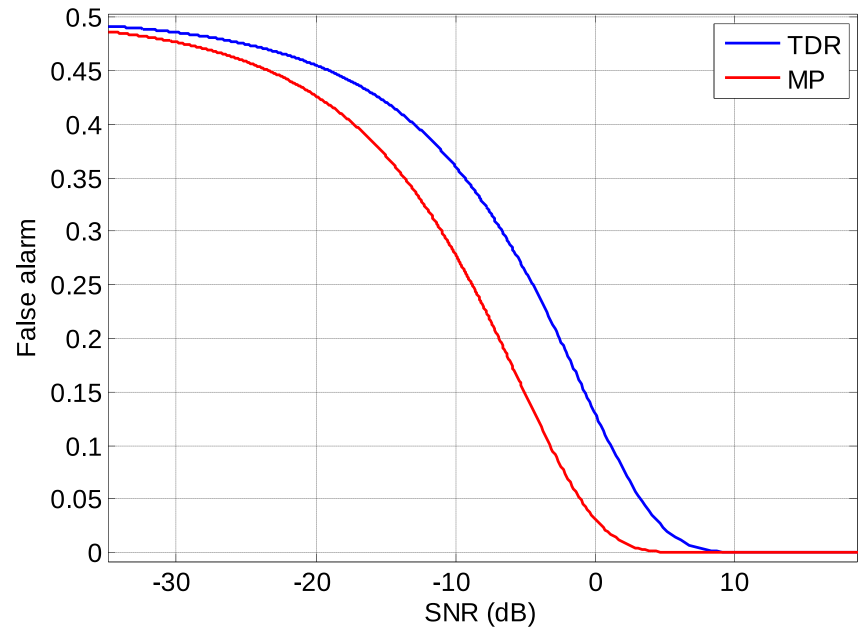
MP fault peak / TDR fault peak = 2.52
MP contrast = 2.81
TDR contrast = 0.78

Experimental results

Detection probability (TP)



False alarm (FA)



Detection threshold : 80% of the fault peak

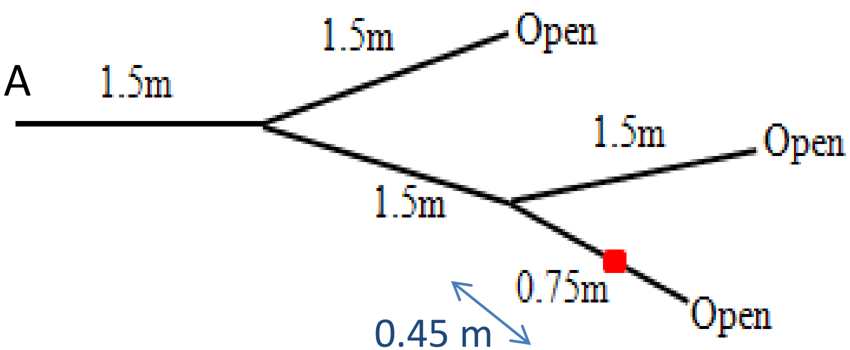
SNR Gain = 4.43 dB

SNR (dB)	Probability Gain (TP)
-10	0.02
0	0.07
10	0.12

SNR (dB)	Probability Gain (FA)
-20	0.03
-10	0.08
0	0.08

Experimental results

Studied network 2



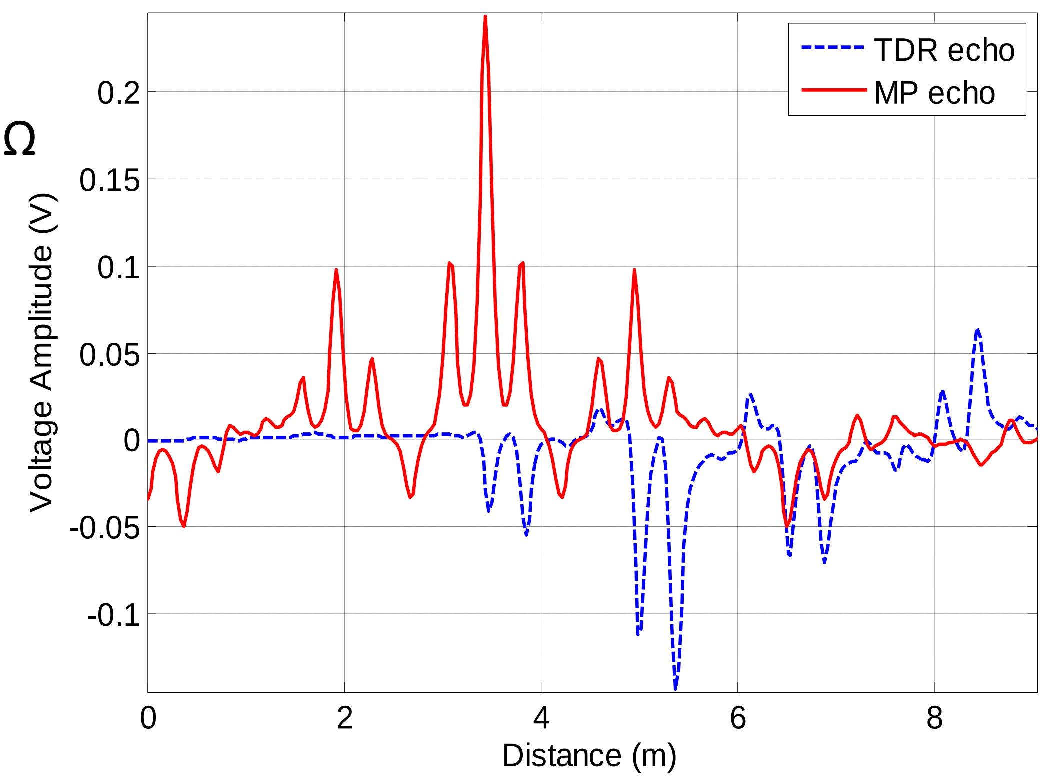
A : injection point, impedance = 50 Ω

Terminations (open circuits)
Coaxial cables
Characteristic impedance : 50 Ω
Velocity of propagation : 0.826 c

Fault : Short circuit

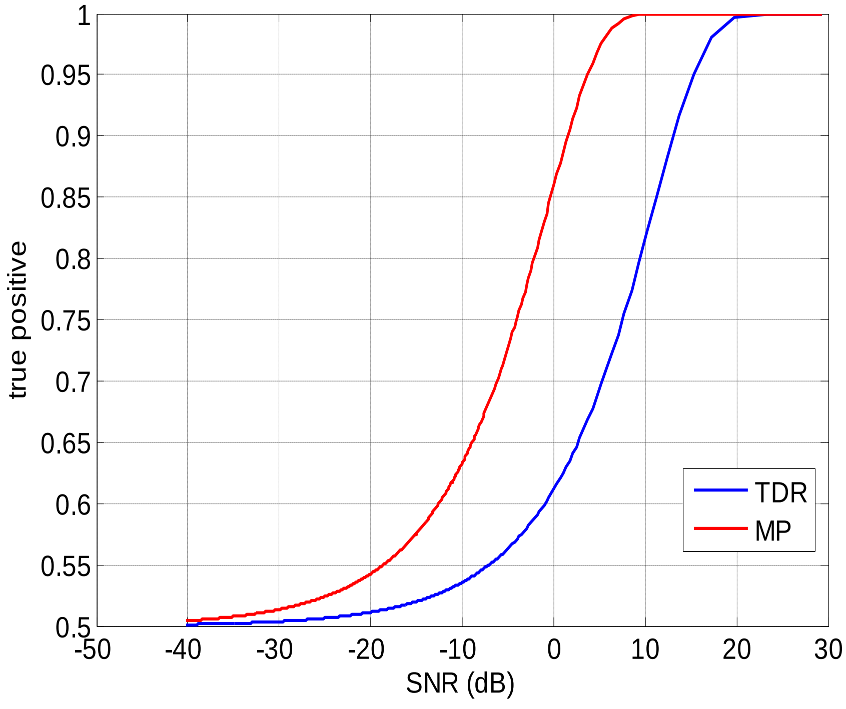
MP and TDR echoes:

MP fault peak / TDR fault peak = 5.87
MP contrast = 2.4
TDR contrast = 0.76
G = 30.38

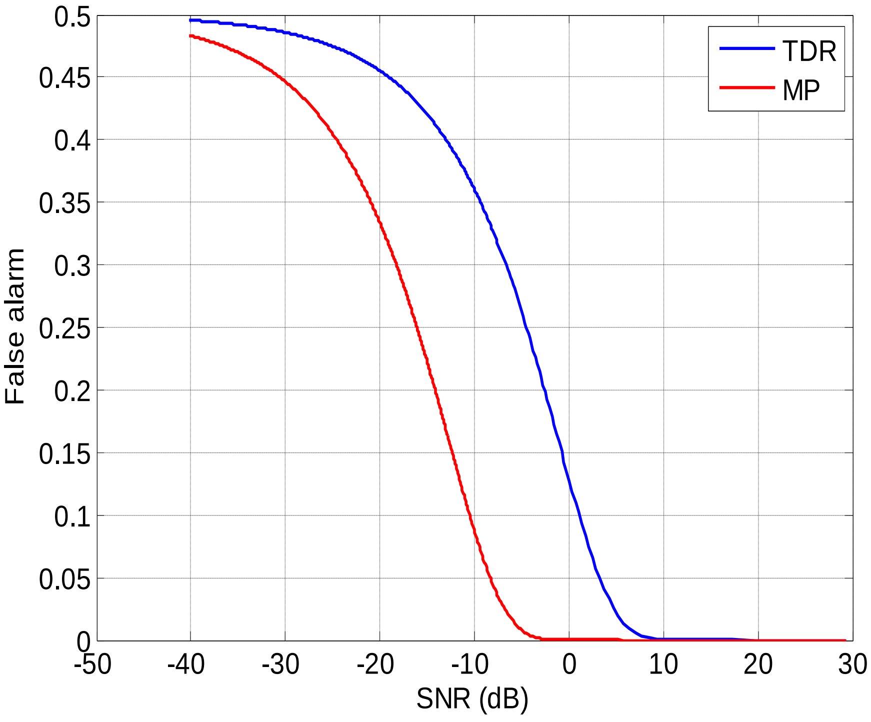


Experimental results

Detection probability



False alarm



SNR Gain = 11.67 dB

SNR (dB)	Probability Gain (TP)
-10	0.1
0	0.24
10	0.18

SNR (dB)	Probability Gain (FA)
-20	0.12
-10	0.27
0	0.13

Conclusion and Perspectives

- The MP approach relies on the idea of adapting the testing signal to the network under test

- It presents major advantages over standard reflectometry techniques in complex configurations

Perspectives

Study fault location based on the idea of synthesizing signals that can focus on the fault location (direct problem), instead of using the inverse problems approach.

Thank You