Practical Solution of Periodic Filtered Approximation as a Convex Quadratic Integer Program

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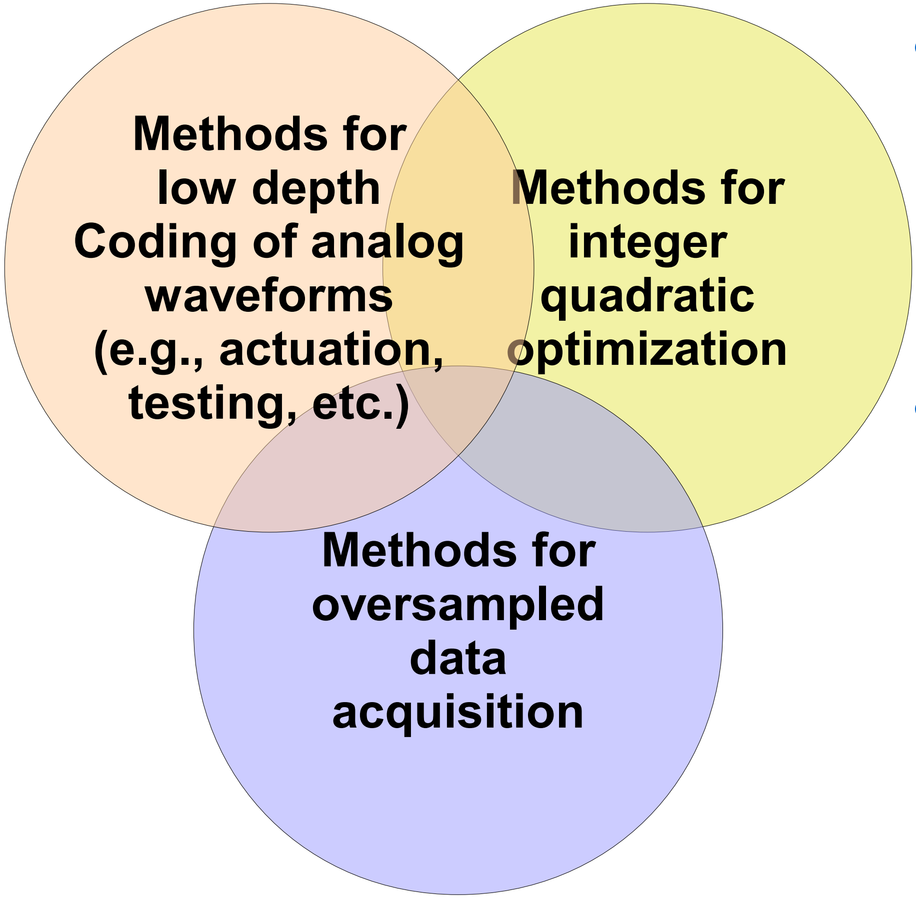
Overview



Example from electronics

- Problems exist where ad-hoc solutions are known
  - Optimization can find new and better solutions
- Reversible
  - Duality: original problem / optimization problem
    - Use optimization to tackle original problem
    - Use techniques from original domain to tackle generic optimization problems

Outline

- Forward path:
  - Attacking a problem from electronics/ actuation/signal-processing by optimization
    - Problem illustration
    - Standard practical solution (PWM modulator)
    - Advanced practical solution (ΔΣ modulator)
    - Issues with practical solutions
    - Re-formalization as an optimization problem
    - Exact and heuristic solution of the optimization problem
    - Experimental evaluation

Outline (ii)

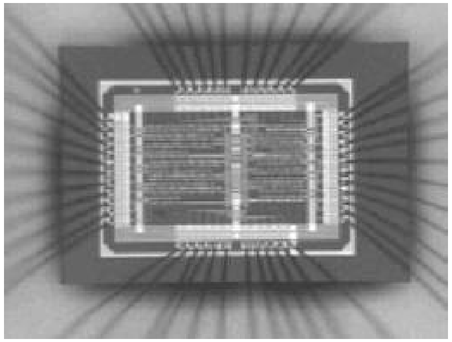
- Backward path
  - Using signal processing to solve optimization problems
    - Duality between C-UDQP optimization problems and P-FA signal processing problems
      - Optimization techniques can be used instead of modulators to solve signal processing problems belonging to the P-FA class
      - Modulators can be used instead of optimization techniques to solve optimization problems belonging to the C-UDQP class
    - Example
    - Towards hardware based solvers

Forward path:

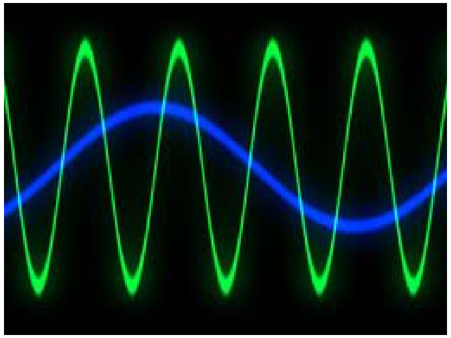
Attacking a problem from electronics by optimization

- We have lived the “digital revolution”, still analog waveforms are needed

Driving AC Motors (synthesis of the AC drive waveforms)



Synthesis of waveforms for Built-in Self Test



Storage of waveforms or algorithmic waveform synthesis

How should them be represented in digital systems?

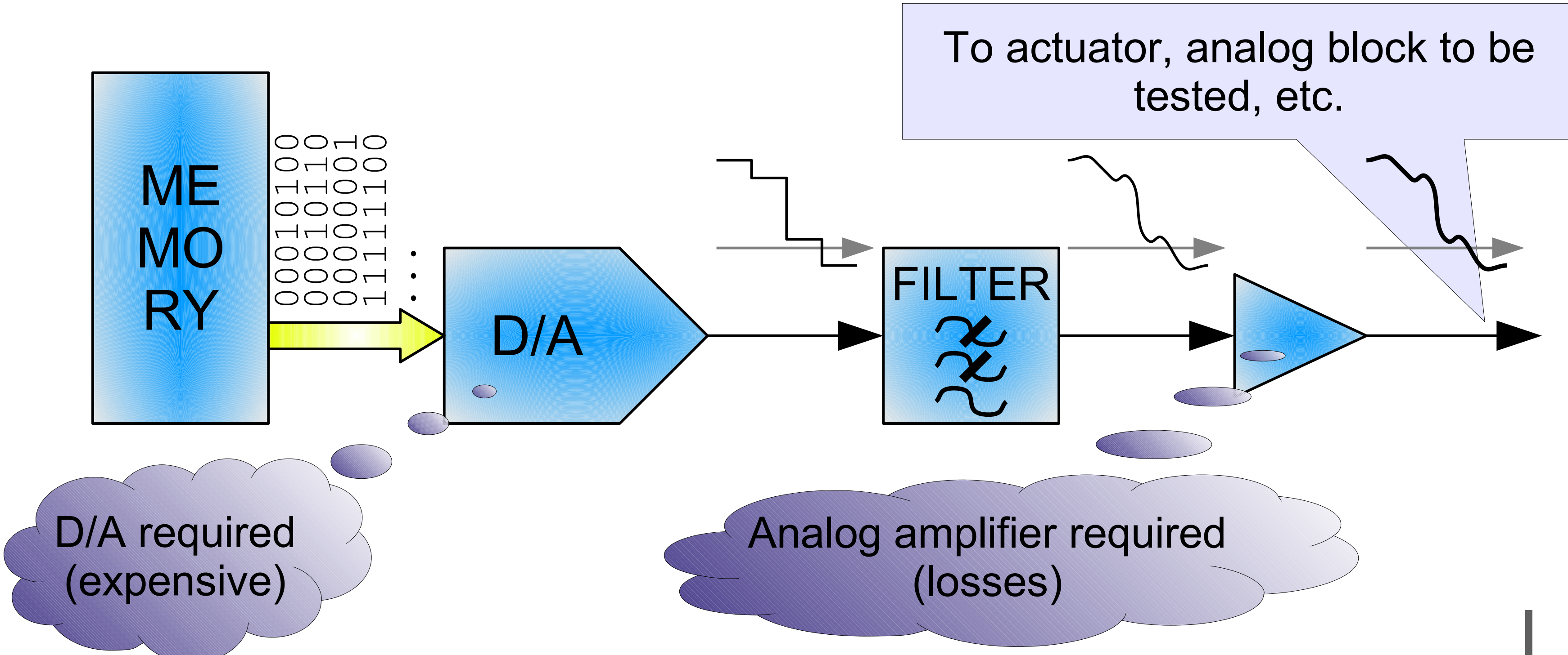
- What is the “best” way of coding them?

Optimisation inherent

Evaluated under a merit factor

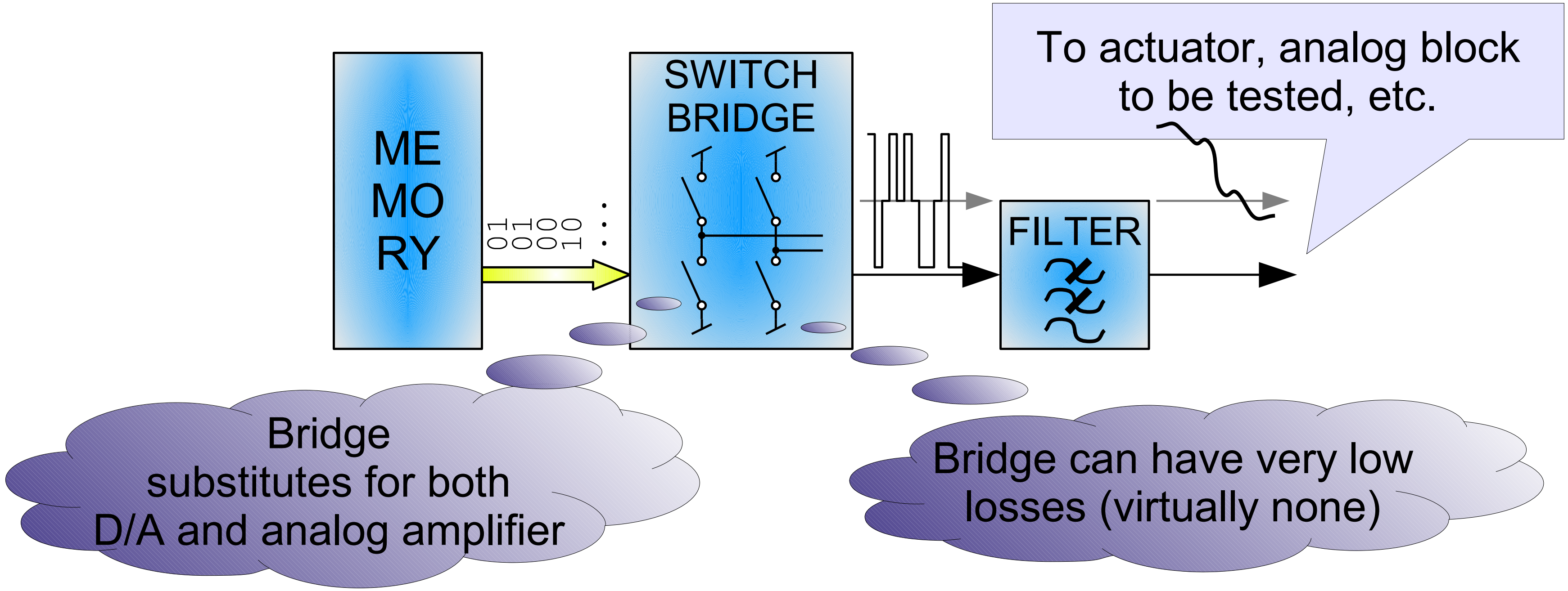
Problem illustration

- Coding an analog waveform
  - Standard solution: PCM (high-depth-code)
    - E.g. cdrom
    - Provably optimal (SNR) under certain constraints (Nyquist)



Problem illustration (ii)

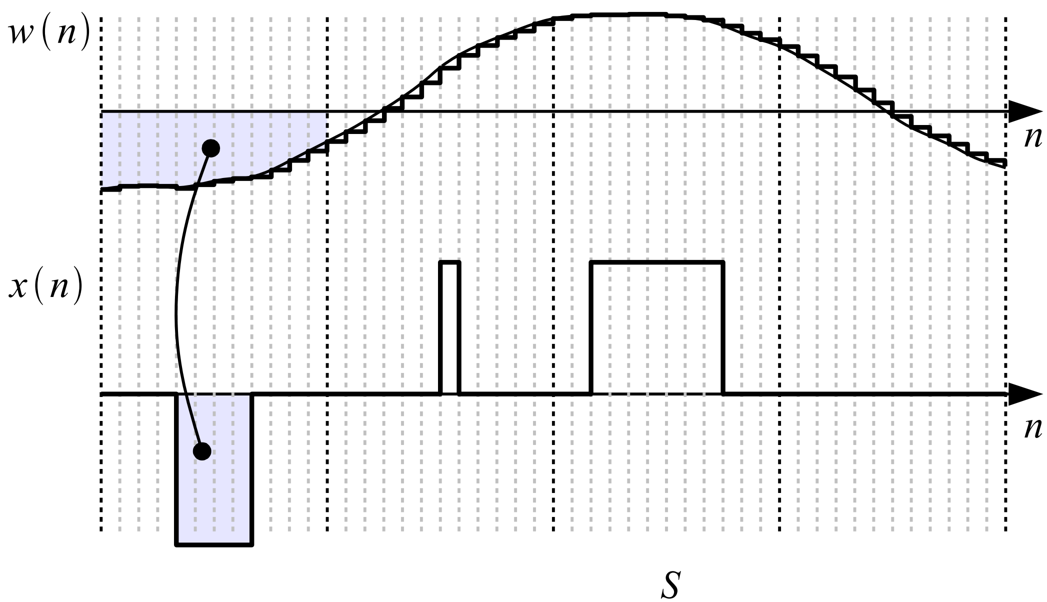
- Low depth codes would be advantageous



  - But how to generate a suitable low depth code?
    - PCM is obvious (Nyquist rate sampling + quantisation), but...
    - … Low depth codes are much less obvious

Standard practical solution (PWM) (i)

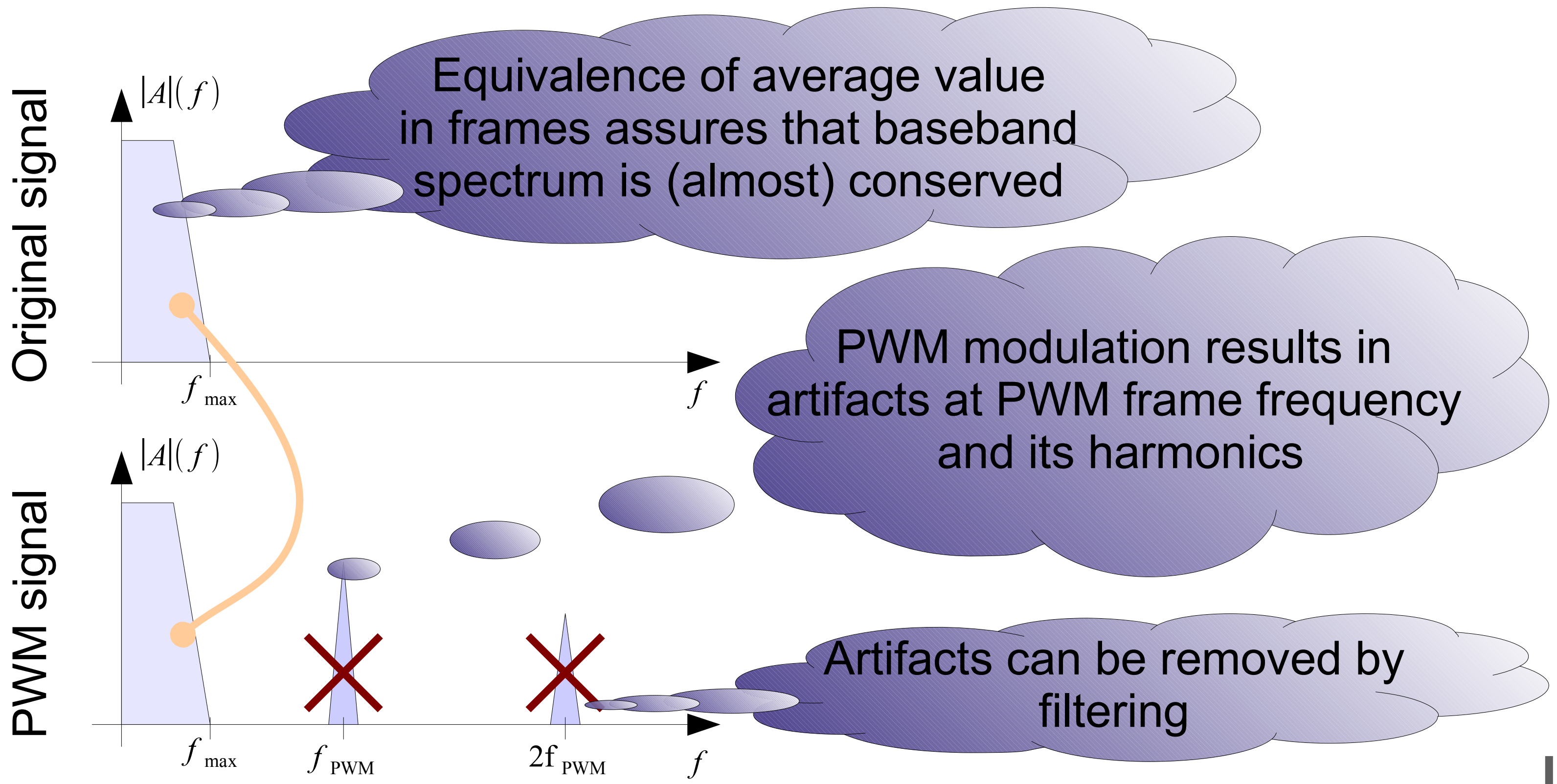
- Divide time axis in frames
  - put in each frame a pulse having the same average value as the original signal



  - Needs a frame rate much higher than the signal bandwidth.

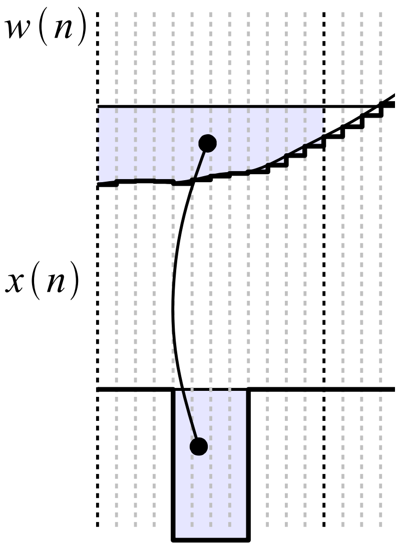
Standard practical solution (PCM) (ii)

- Key idea in PWM
  - In the frequency domain:



Advanced practical solution (PWM) (iii)

- Key issue of PWM:
  - In digital implementations PWM pulses need to be aligned with a reference clock



- Loss in resolution unless clock frequency is very high
  - E.g.:

BAD
clock freq vs
SNR
trade-off

$$f_{CLK} = 2^{10} f_{PWM} \approx 2^{10} 100 f_{max}$$

$$f_{max} = 20 \text{ kHz} \Rightarrow f_{CLK} \approx 2 \text{ GHz}$$

!!!

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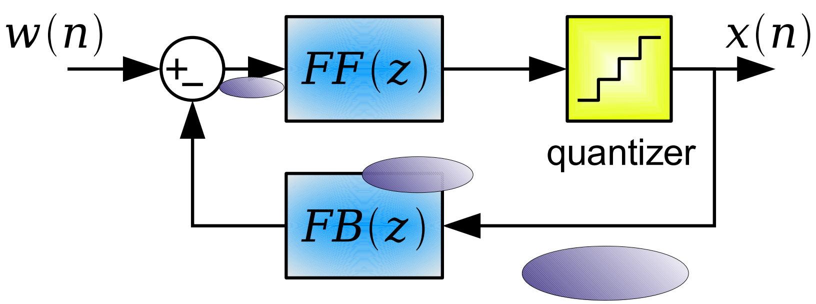
Advanced practical solution (ΔΣ) (i)

- Use a modulator structure attempting to “zero” the approximation error (closed loop controller)

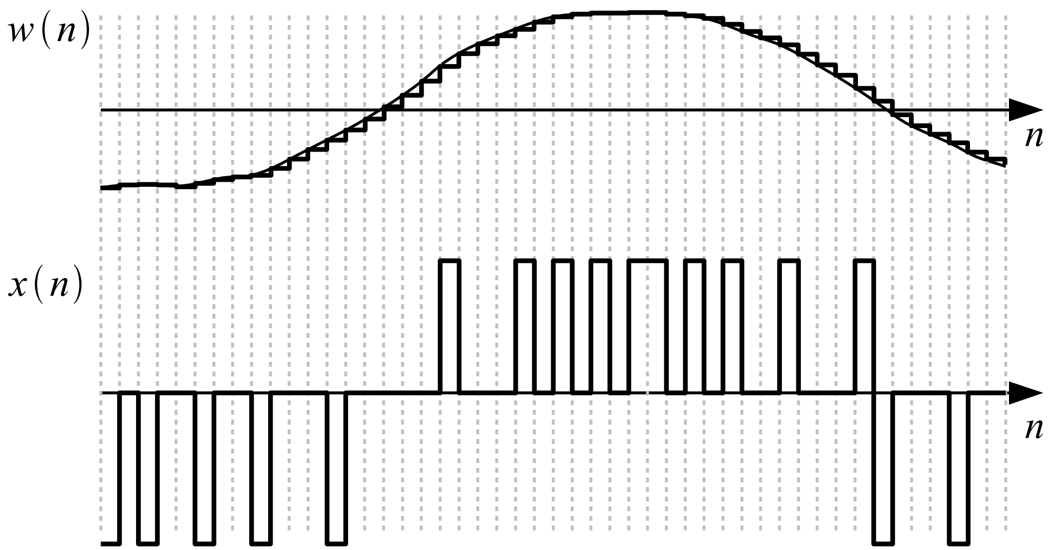


Delta-Sigma modulator

If *FF(z)* is large (at some frequencies)
then the error is low
(at those frequencies)

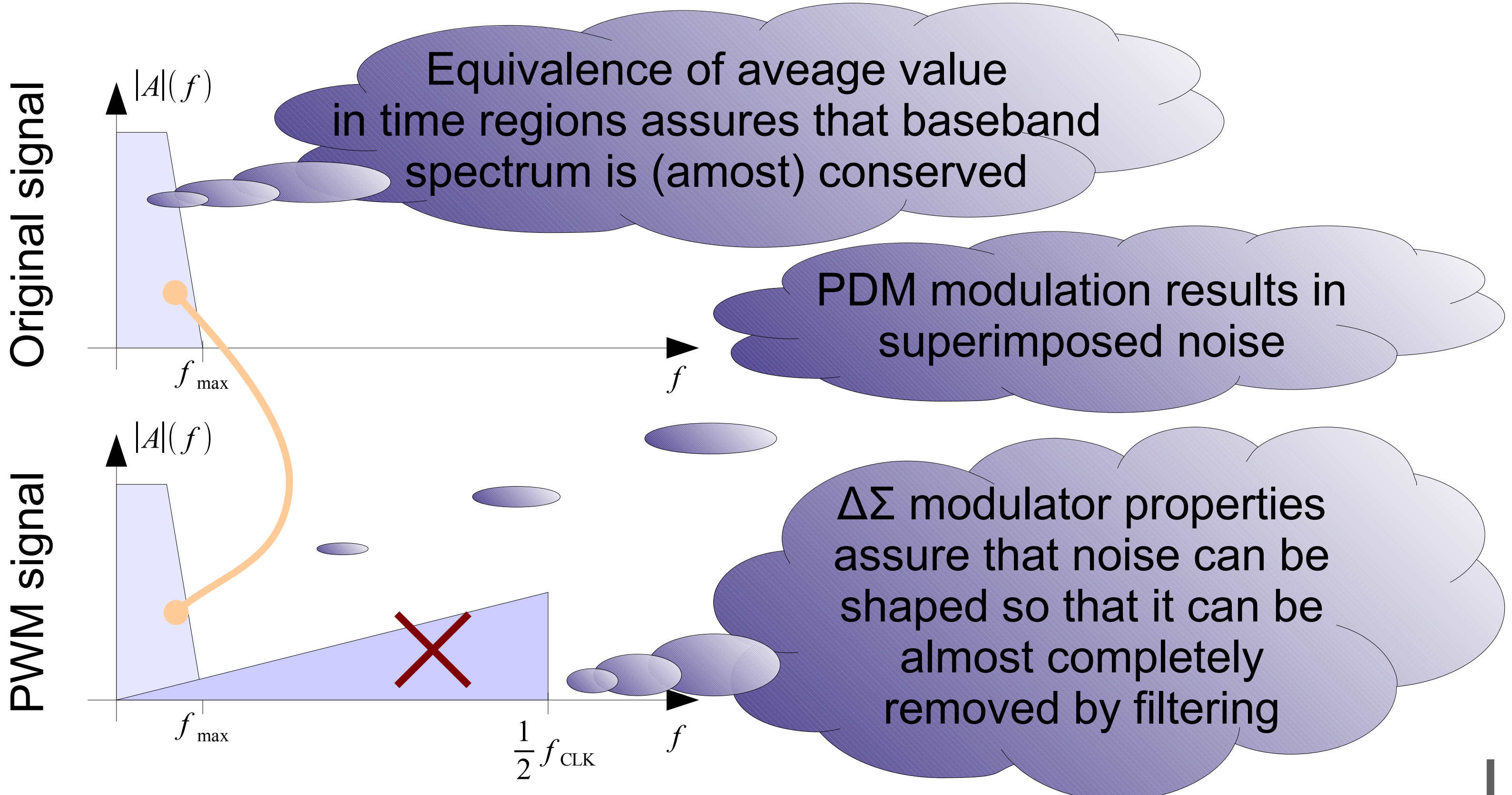
Advanced practical solution (ΔΣ) (ii)

- Result is a Pulse Density Modulation (PDM)
  - Again, coded signal at any region has an average value approximately equal to that of the original signal



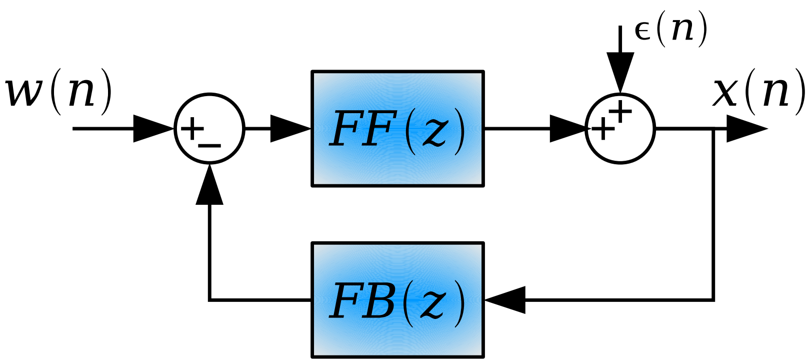
Advanced practical solution (ΔΣ) (iii)

- Key idea of ΔΣ
  - In the frequency domain:



Advanced practical solution (ΔΣ)

- Can be explained by approximated linear model



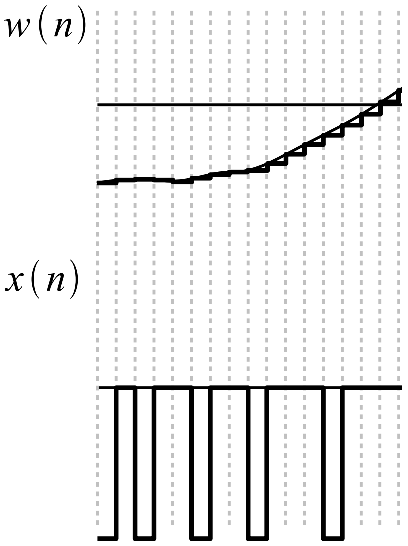
$$\begin{cases} STF(z) = \dfrac{FF(z)}{1+FF(z)\,FB(z)} \\ NTF(z) = \dfrac{1}{1+FF(z)\,FB(z)} \end{cases}$$

Signal transfer function
take it unitary

Noise transfer function
take it so that noise is pushed
out of the way
(typically at high frequencies)

Advanced practical solution (ΔΣ) (v)

- Key advantage
  - Clock rate can be much lower in comparison to PWM
- Key issues:
  - Switch rate is high, design relies on approximations



$$\overline{f_{\mathrm{SWITCH}}} \approx \alpha f_{\mathrm{CLK}} = \alpha f_{\mathrm{MAX}} OSR$$

$$\alpha \in [0.2, 0.8] \quad OSR \gg 64\,(\mathrm{typ})$$

Keeping OSR low would be helpful. But OSR cannot be pushed too low without compromising on quality.

Issues with the “practical” solutions

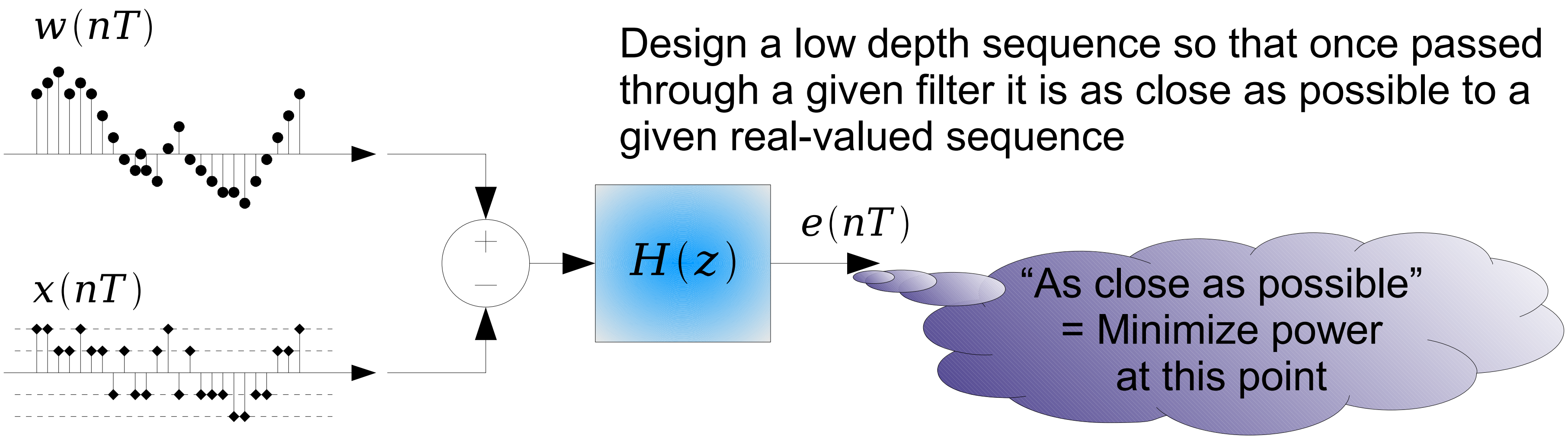
- Both PWM and ΔΣ work in the task
  - Proved by many successful deployments in industry
- But both have not completely resolved issues

WHY?

- Operation is a **side effect** of the modulator properties
  - Be it the PWM or the ΔΣ modulator
- not the result of an explicit optimization effort

Re-formalization as an optimization problem (i)

- In signal processing the problem to be solved is called **Filtered Approximation (FA)**



Design a low depth sequence so that once passed through a given filter it is as close as possible to a given real-valued sequence

  – To convert the problem in an optimisation problem, the sequence to design must be finite-length
    - Work on signal windows or assume **periodicity**

Re-formalization as an optimization problem (ii)

- Assume that target signal is **periodic**
  - OK for most actuation and testing applications
    - Typically relying on sine-waves or combinations of sinewaves
- Problem is now called **Periodic Filtered Approximation (P-FA)**

with
$N \in \mathbb{N}$ number of clock cycles in each period
$\boldsymbol{w} \in \mathbb{R}^N$ vector holding the target signal
$\boldsymbol{x} \in \mathcal{A}^N$ vector of pulses to be optimized

E.g. $\mathcal{A} = \{-1, 0, 1\}$

- Average error power per period E is:
  - Quadratic form in $\boldsymbol{x}$
    - Positive Definite + other interesting properties

Re-formalization as an optimization problem (iii)

$$E = \boldsymbol{x}^T \boldsymbol{Q}\,\boldsymbol{x} + \boldsymbol{L}^T \boldsymbol{x} + c$$

$$\boldsymbol{Q} \text{ s.t. } q_{j,k} = \frac{1}{N}\sum_{i=0}^{N-1} \left| H\left(e^{i 2\pi \frac{i}{N}}\right)\right|^2 e^{i 2\pi i \frac{(j-k)}{N}} \qquad \boldsymbol{L} = -2\,\boldsymbol{w}^T \boldsymbol{Q} \qquad c = \boldsymbol{w}^T \boldsymbol{Q}\,\boldsymbol{w}$$

$$\boldsymbol{Q} = \begin{pmatrix} q_0 & q_1 & q_2 & q_3 & \cdots & q_{N-1} \\ q_{N-1} & q_0 & q_1 & q_2 & \cdots & q_{N-2} \\ q_{N-2} & q_{N-1} & q_0 & q_1 & \cdots & q_{N-3} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ q_1 & q_2 & q_3 & q_4 & \cdots & q_0 \end{pmatrix}$$

$\boldsymbol{Q}$ is circulant!

- Optimization problem is

$$\operatorname*{minarg}_{\boldsymbol{x} \in \mathcal{A}^N} \boldsymbol{x}^T \boldsymbol{Q}\,\boldsymbol{x} + \boldsymbol{L}^T \boldsymbol{x}$$

(Circulant) Unconstrained Discrete Quadratic Programming

Exact and Heuristic solution of the optimization problem

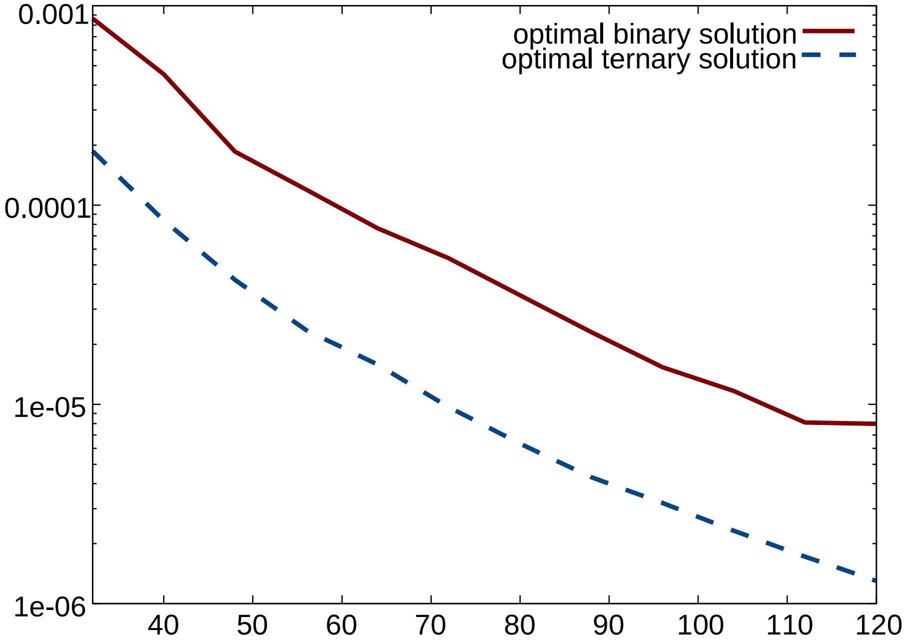
- Optimization problem can now be solved by
  - Exact method (branch and bound)
  - Heuristic method (genetic, etc.)
- Why isn't this approach mainstream
  - Electronic people like thinking in terms of electronic primitives (modulators)
  - Optimization is a computation intensive task
    - Only problems up to a certain dimension can be solved exactly
      - today improvements in algorithms + much greater computation power is available

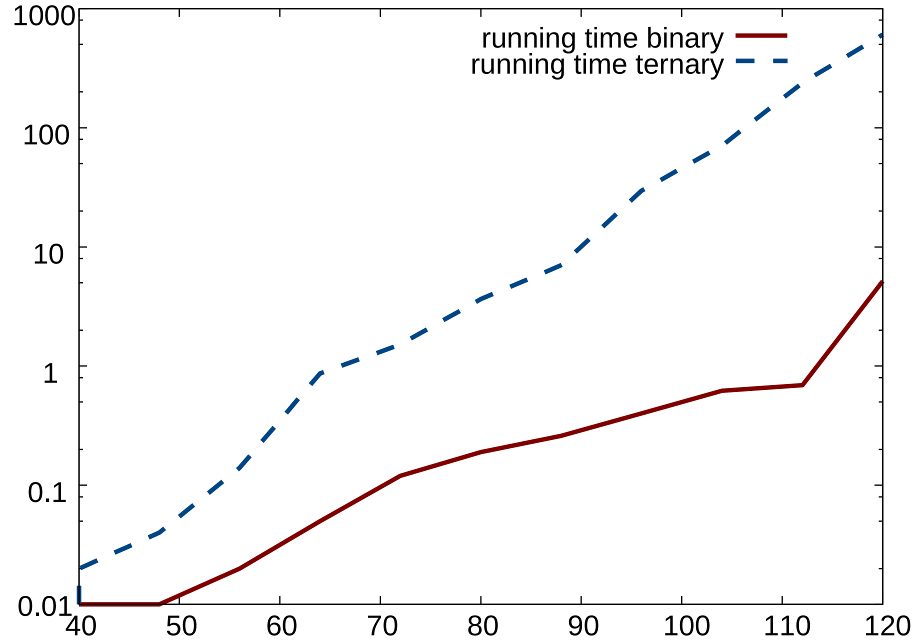
Experimental validation (i)

- Exact algorithm [Buchheim, Lodi, Caprara 2010]
    - Branch and bound taking advantage of the “low depth” nature of the problem
    - Started by a genetic algorithm [Lodi 1999]
- Heuristic algorithm
    - Branch and bound limited in execution time
    - Since it internally employs a genetic algorithm at most it falls back to a genetic approach
- Comparison to ΔΣ
    - That can be re-interpreted as another heuristic approach for the optimization problem.

Experimental validation (ii)

- Accuracy and computation time of the exact solver for the binary and ternary case

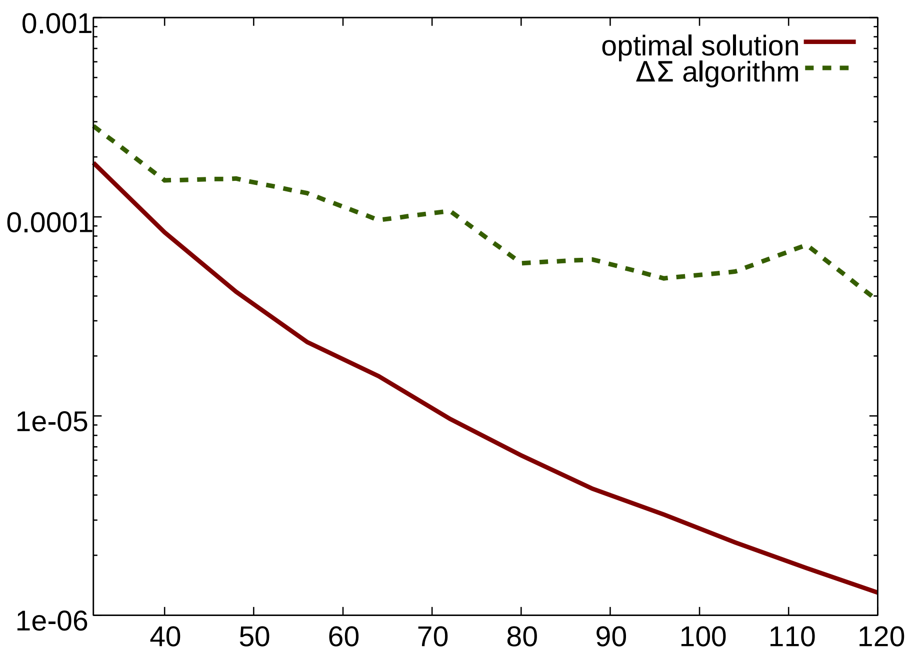


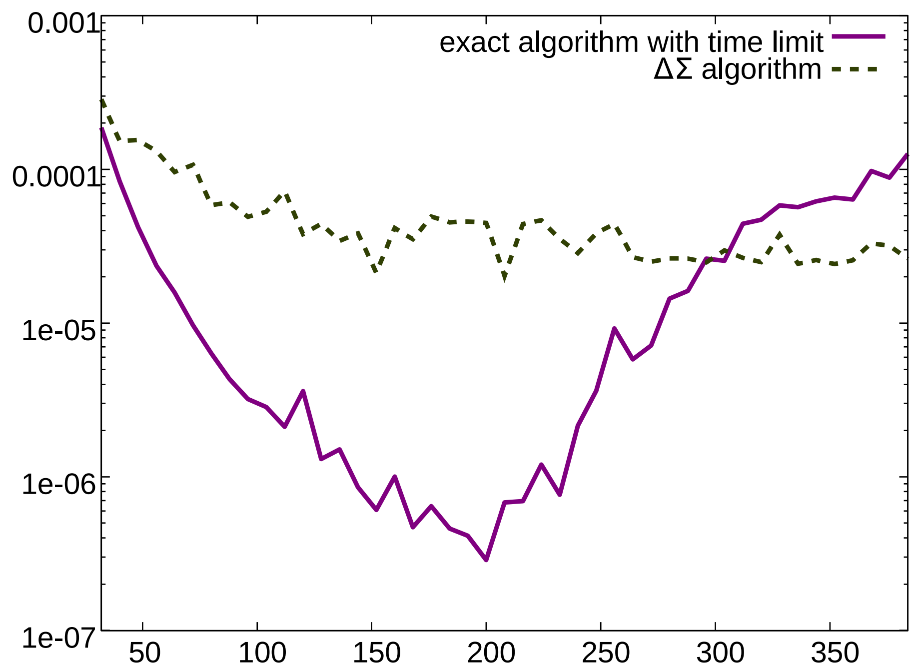


- Problem is solvable for N up to about 150

Experimental validation

- Comparison of exact and heuristic algorithms





  - Gap between current practical solution and optimum
  - Conventional heuristics are beaten by ΔΣ for very large problems

Experimental validation (iv)

- To summarize (up to this point)
  - Exact optimization approach can have a large advantage for small size problems
  - Conventional heuristic optimization approaches can have some advantage for mid-size problems
  - ΔΣ is so far the “best heuristic” for very large problems
    - However, optimizers can potentially improve by specializing them more on the circulant form
  - Optimization is interesting also because it is more flexible
    - E.g., it could tackle new merit factors taking into account other “qualities” (power efficiency, switch misbehaviours, etc.)

Backward path:

Using signal processing to solve optimization problems

- The approach illustrated so far shows that there is a duality between P-FA problems and C-UDQP problems
- This duality can be used backwards
  - Take an optimization problem that belongs to the C-UDQP class
  - Convert it into a P-FA problem
  - Use a ΔΣ modulator to solve it
    - i.e., use ΔΣ modulation as a heuristic solution approach in place of more conventional heuristics (evolutionary, etc.)

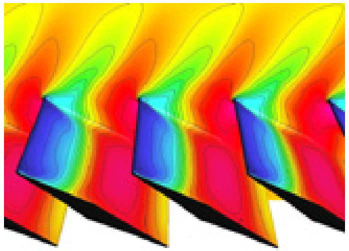
Duality between CUDQP optimization problems and P-FA signal processing problems

- From: $\underset{x \in \mathcal{A}^N}{\text{minarg}}\, \boldsymbol{x}^T \boldsymbol{Q}\, \boldsymbol{x} + \boldsymbol{L}^T \boldsymbol{x}$
- If $\boldsymbol{Q}$ is circulant and positive definite
  – Define input signal and filter for P-FA problem
  – Use ΔΣ modulation to generate a solution where "noise is out of the way" wrt the P-FA filter
- Why doing so?
  – Previous tests suggest that it may be an interesting heuristic for very large problems
  – Duality between P-FA and C-UDQP is an intriguing topic from a theoretical point of view

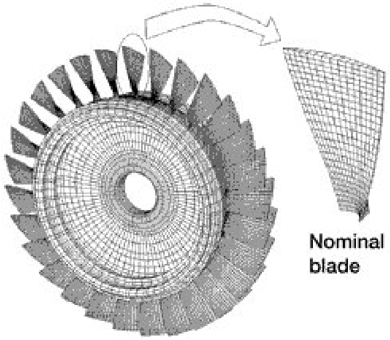
Example

- Approach validated on real-world problems
  - Small ones, so a benchmark exists

Turbomachines: require compressor stages i.e., fans with multiple blades



Flutter:unstable blade vibrations by coupling between aerodynamics and blade mechanics

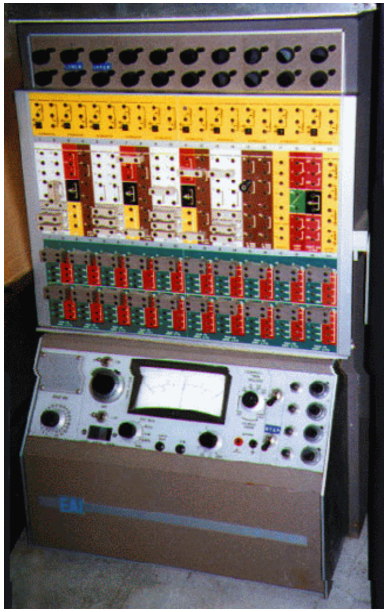


Can be controlled by artificially mistuning the blades [Shaphiro]

- Optimization of mistuning is under certain conditions a C-UDQP problem
  - Solution with ΔΣ modulators presented at ISCAS 2010

Towards hardware based solvers

- In a recent past it was common to have analog computers
  - Machines solving problems by having their hardware match the problem to be solved, not doing numeric computations about it



Electronic Associates Inc. “desktop” analog computer (1964)

Can ΔΣ modulators be XXI century hardware based solvers for large optimization problems belonging to specific classes?

E.g. transform optimization problem in signal processing problem by some CAD tool + program flexible modulator based co-processor to solve problem in hardware

Conclusions

- It makes sense to try to tackle by explicit optimization problems that engineers typically tackle by other means
  - Improvements in the way in which we solve problems
- In some cases it may lead to the discovery of dualities
- Which open intriguing opportunities
  - Improvements in the way in which we do heuristic optimization?

Thanks for attending!
I'd be pleased to answer your questions

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