Artificial Intelligence Techniques for Automatic Reformulation and Solution of Structured Mathematical Models

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- In the application of solution methods we have the same phenomenon.

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Definition of an environment that allows

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I-DARE - Intelligence–Drive Automatic Reformulation Engine.

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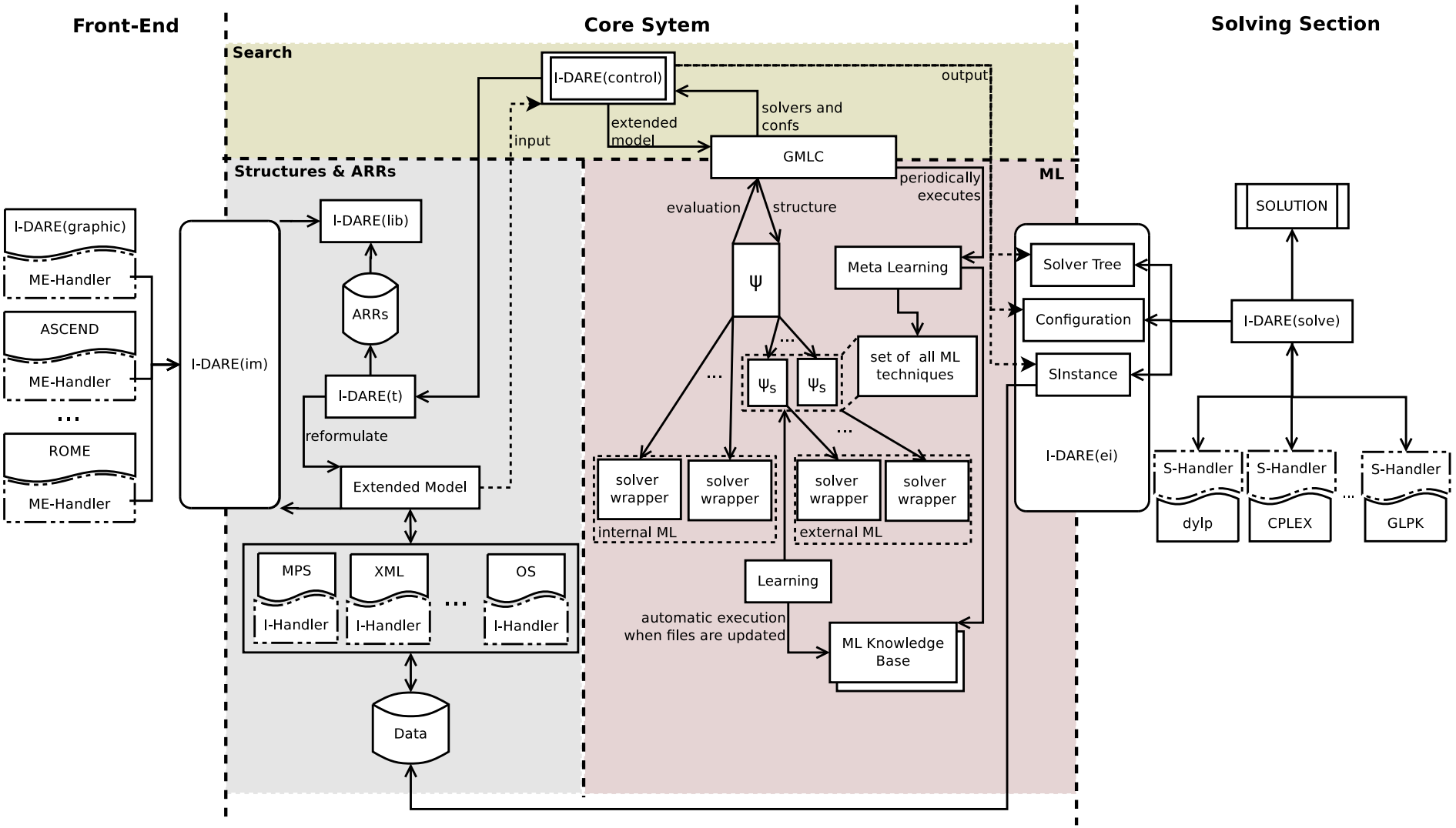
Front-End
Core Sytem
Solving Section
Search
I-DARE(control)
output
solvers and confs
extended model
input
GMLC
Structures & ARRs
periodically executes
ML
evaluation
structure
I-DARE(graphic)
ME-Handler
ASCEND
ME-Handler
ROME
ME-Handler
I-DARE(im)
I-DARE(lib)
ARRs
I-DARE(t)
reformulate
Extended Model
Ψ
Meta Learning
Ψs
Ψs
set of all ML techniques
solver wrapper
solver wrapper
internal ML
solver wrapper
solver wrapper
external ML
Solver Tree
Configuration
SInstance
I-DARE(ei)
SOLUTION
I-DARE(solve)
S-Handler
S-Handler
S-Handler
dylp
CPLEX
GLPK
MPS
XML
OS
I-Handler
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Learning
automatic execution when files are updated
ML Knowledge Base
Data

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- All solver information is accessible from $\mathcal{F}$LORA-*2* (registration, configuration and signatures), plus the solvers' tree structure.
- Configurations, the used XML defines a well structured way to access the solver's configuration.
  - We can access the parameters and their domains.

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- and returns an equivalent instance, plus the solver's tree and their configurations.
- We focused on the subproblem of searching in the (Solver, Configuration) space.

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- An arithmetic or algorithmic description of ψ will hardly be available.
- We propose the application of ML techniques to approximate it (based on known observations).

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- Relevant "features" may change from problem class to problem class, even more, from solver to solver.
  - for LP: number of variables, constraints, density of the constraint matrix.
  - degeneracy of the vertex of the polyhedron affects Simplex but not Interior-Point.
- Each solver must supply the set of features relevant to itself.

Features

- A solver wrapper will provide with a set of features relevant to the solver and possible configurations.

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- If it is external it returns the features and configuration to be evaluated by a ML technique.
- There intermediate scenarios (also external) that may use sophisticated techniques to compute the features.

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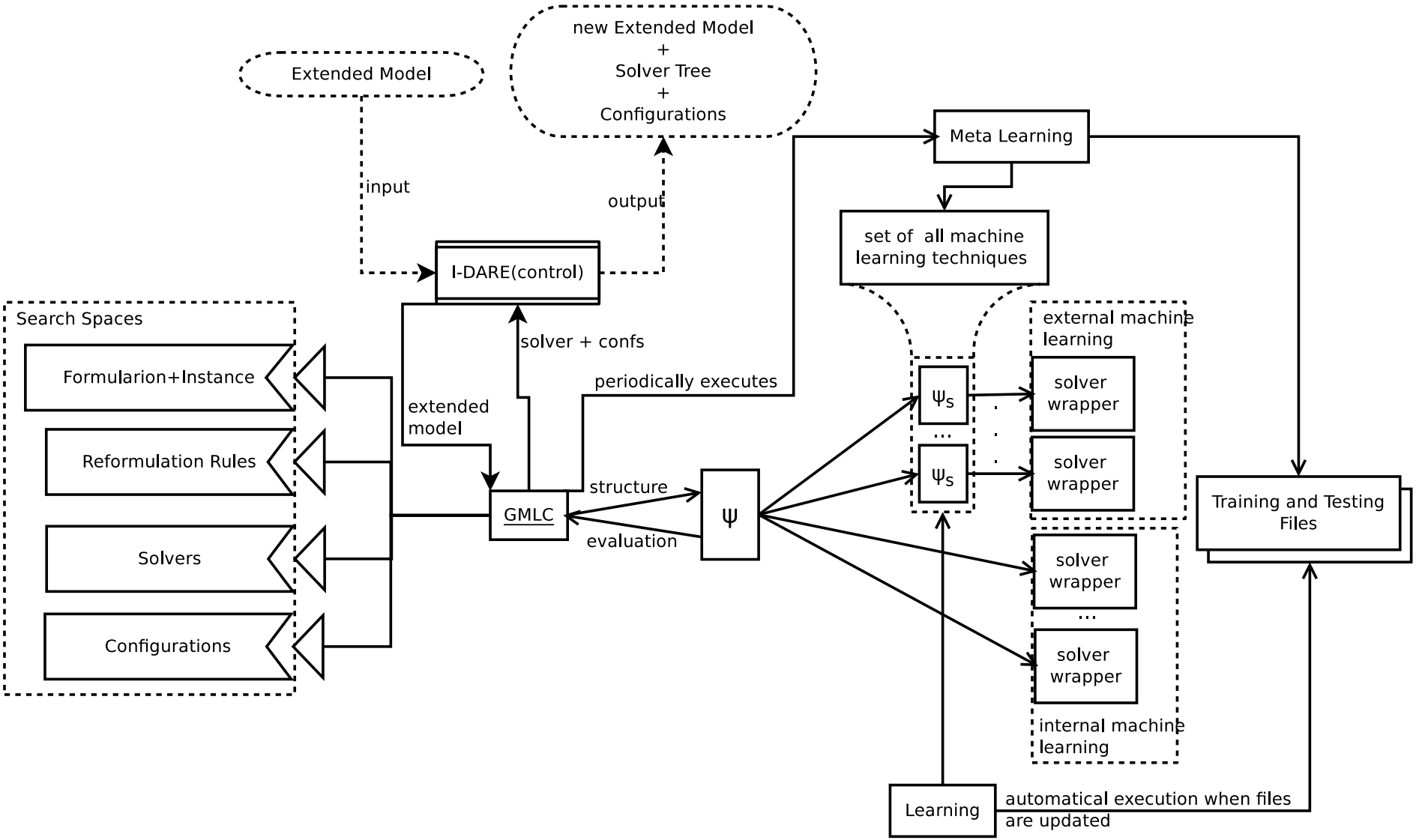
- ML will provide with sufficient tools to handle the search in the (Solver, Configuration) subspace.

The overall process



Experiments – MCF

Number of graphs: 144

Solver	nbest	b/d ratio	% best
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s. err	b/ml ratio	ml/dR ratio	ml/dS ratio
0.145	0.787 (0.208)	0.933 (0.342)	0.791 (0.450)

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  - selection of solver + configuration can be done automatically,
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  - we may use potentially any search mechanism to move in the reformulation space

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- Application of different search strategies to deal with the reformulation space.